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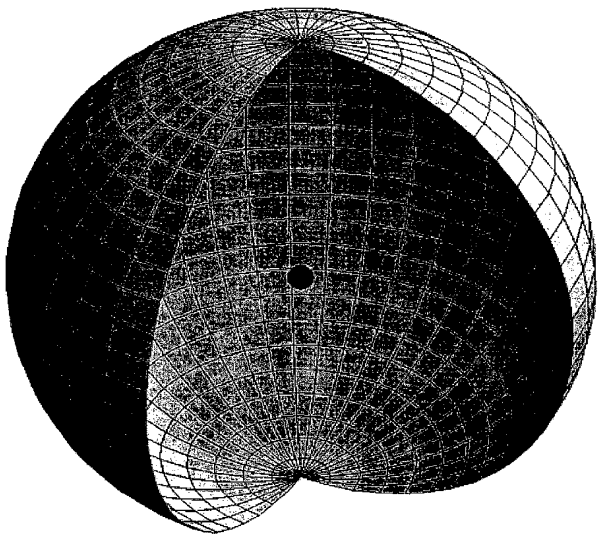
(54) Title: METHOD AND SYSTEM OF COMPUTING AND RENDERING THE NATURE OF ATOMS AND ATOMIC IONS



(57) Abstract: A method and system of physically solving the charge, mass, and current density functions of atoms and atomic ions using Maxwell's equations and computing and rendering the nature of bound using the solutions. The results can be displayed on visual or graphical media. The display can be static or dynamic such that electron spin and rotation motion can be displayed in an embodiment. The displayed information is useful to anticipate reactivity and physical properties. The insight into the nature of bound electrons can permit the solution and display of other atoms and atomic ions and provide utility to anticipate their reactivity and physical properties.

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METHOD AND SYSTEM OF COMPUTING AND RENDERING THE NATURE OF ATOMS AND ATOMIC IONS

This application claims priority to U.S. Provisional Appl'n Ser. Nos. 60/542,278, filed February 9, 2004, and 60/534,112, filed January 5, 2004, the complete disclosures of which are incorporated herein by reference.

This application also claims priority to U.S. Provisional Appl'n entitled "The Grand Unified Theory of Classical Quantum Mechanics" filed January 3, 2005, attorney docket No. 62226-BOOK1, the complete disclosure of which is incorporated herein by reference.

1. Field of the Invention

This invention relates to a method and system of physically solving the charge, mass, and current density functions of atoms and atomic ions and computing and rendering the nature of these species using the solutions. The results can be displayed on visual or graphical media. The displayed information is useful to anticipate reactivity and physical properties, as well as for educational purposes. The insight into the nature of bound electrons can permit the solution and display of other atoms and ions and provide utility to anticipate their reactivity and physical properties.

2. Background of the Invention

While it is true that the Schrödinger equation can be solved exactly for the hydrogen atom, the result is not the exact solution of the hydrogen atom since electron spin is missed entirely and there are many internal inconsistencies and nonphysical consequences that do not agree with experimental results. The Dirac equation does not reconcile this situation. Many additional shortcomings arise such as instability to radiation, negative kinetic energy states, intractable infinities, virtual particles at every point in space, the Klein paradox, violation of Einstein causality, and "spooky" action at a distance. Despite its successes, quantum mechanics (QM) has remained mysterious to all who have encountered it. Starting with Bohr and progressing into the present, the departure from intuitive, physical reality has widened. The connection between quantum mechanics and reality is more than just a "philosophical" issue. It reveals that quantum mechanics is not a correct or complete theory of the physical world and that inescapable internal inconsistencies

and incongruities arise when attempts are made to treat it as a physical as opposed to a purely mathematical "tool". Some of these issues are discussed in a review by Laloë [Reference No. 1]. But, QM has severe limitations even as a tool. Beyond one-electron atoms, multielectron-atom quantum mechanical equations can not be solved except by approximation methods involving adjustable-parameter theories (perturbation theory, variational methods, self-consistent field method, multi-configuration Hartree Fock method, multi-configuration parametric potential method, $1/Z$ expansion method, multi-configuration Dirac-Fock method, electron correlation terms, QED terms, etc.)—all of which contain assumptions that can not be physically tested and are not consistent with physical laws. In an attempt to provide some physical insight into atomic problems and starting with the same essential physics as Bohr of e^- moving in the Coulombic field of the proton and the wave equation as modified after Schrödinger, a classical approach was explored which yields a model which is remarkably accurate and provides insight into physics on the atomic level [2-4].

Physical laws and intuition are restored when dealing with the wave equation and quantum mechanical problems. Specifically, a theory of classical quantum mechanics (CQM) was derived from first principles that successfully applies physical laws on all scales. Rather than use the postulated Schrödinger boundary condition: "$\Psi \to 0$ as $r \to \infty$", which leads to a purely mathematical model of the electron, the constraint is based on experimental observation. Using Maxwell's equations, *the classical wave equation is solved with the constraint that the bound* $n = 1$ *-state electron cannot radiate energy*. The electron must be extended rather than a point. On this basis with the assumption that physical laws including Maxwell's equation apply to bound electrons, the hydrogen atom was solved exactly from first principles. The remarkable agreement across the spectrum of experimental results indicates that this is the correct model of the hydrogen atom. In the present invention, the physical approach was applied to multielectron atoms that were solved exactly disproving the deep-seated view that such exact solutions can not exist according to quantum mechanics. The general solutions for one through twenty-electron atoms are given. The predictions are in remarkable agreement with the experimental values known for 400 atoms and ions.

Classical Quantum Theory of the Atom Based on Maxwell's Equations

The old view that the electron is a zero or one-dimensional point in an all-space probability wave function $\Psi(x)$ is not taken for granted. The theory of classical quantum mechanics (CQM), derived from first principles, must successfully and consistently apply physical laws on all scales [2-7]. Historically, the point at which QM broke with classical laws can be traced to the issue of nonradiation of the one electron atom that was addressed by Bohr with a postulate of stable orbits in defiance of the physics represented by Maxwell's equations [2-9]. Later physics was replaced by "pure mathematics" based on the notion of the inexplicable wave-particle duality nature of electrons which lead to the Schrödinger equation wherein the consequences of radiation predicted by Maxwell's equations were ignored. Ironically, both Bohr and Schrödinger used the electrostatic Coulomb potential of Maxwell's equations, but abandoned the electrodynamic laws. Physical laws may indeed be the root of the observations thought to be "purely quantum mechanical", and it may have been a mistake to make the assumption that Maxwell's electrodynamic equations must be rejected at the atomic level. Thus, in the present approach, the classical wave equation is solved with the constraint that a bound $n = 1$-state electron cannot radiate energy.

Thus, herein, derivations consider the electrodynamic effects of moving charges as well as the Coulomb potential, and the search is for a solution representative of the electron wherein there is acceleration of charge motion without radiation. The mathematical formulation for zero radiation based on Maxwell's equations follows from a derivation by Haus [16]. The function that describes the motion of the electron must not possess spacetime Fourier components that are synchronous with waves traveling at the speed of light. Similarly, nonradiation is demonstrated based on the electron's electromagnetic fields and the Poynting power vector.

It was shown previously [2-6] that CQM gives closed form solutions for the atom including the stability of the $n = 1$ state and the instability of the excited states, the equation of the photon and electron in excited states, the equation of the free electron, and photon which predict the wave particle duality behavior of particles and light. The current and charge density functions of the electron may be directly physically interpreted. For example, spin angular momentum results from the

motion of negatively charged mass moving systematically, and the equation for angular momentum, $\mathbf{r} \times \mathbf{p}$, can be applied directly to the wave function (a current density function) that describes the electron. The magnetic moment of a Bohr magneton, Stern Gerlach experiment, g factor, Lamb shift, resonant line width and shape, selection rules, correspondence principle, wave particle duality, excited states, reduced mass, rotational energies, and momenta, orbital and spin splitting, spin-orbital coupling, Knight shift, and spin-nuclear coupling, and elastic electron scattering from helium atoms, are derived in closed form equations based on Maxwell's equations. The calculations agree with experimental observations. In contrast to the failure of the Bohr theory and the nonphysical, adjustable-parameter approach of quantum mechanics, the nature of the chemical bond is given in exact solutions of hydrogen molecular ions and molecules that match the data for 26 parameters [3]. In another published article, rather than invoking renormalization, untestable virtual particles, and polarization of the vacuum by the virtual particles, the results of QED such as the anomalous magnetic moment of the electron, the Lamb Shift, the fine structure and hyperfine structure of the hydrogen atom, and the hyperfine structure intervals of positronium and muonium (thought to be only solvable using QED) are solved exactly from Maxwell's equations to the limit possible based on experimental measurements [6].

In contrast to short comings of quantum mechanical equations, with CQM, multielectron atoms can be exactly solved in closed form. Using the nonradiative wave equation solutions that describe the bound electron having conserved momentum and energy, the radii are determined from the force balance of the electric, magnetic, and centrifugal forces that corresponds to the minimum of energy of the system. The ionization energies are then given by the electric and magnetic energies at these radii. One through twenty-electron atoms are solved exactly except for nuclear hyperfine structure effects of atoms other than hydrogen. (The spreadsheets to calculate the energies are available from the internet [17]). For 400 atoms and ions the agreement between the predicted and experimental results are remarkable.

Using the same unique physical model for the two-electron atom in all cases, it was confirmed that the CQM solutions give the accurate model of atoms and ions by solving conjugate parameters of the free electron, ionization energy of helium and all two electron atoms, electron scattering of helium for all angles, and all He I

excited states as well as the ionization energies of multielectron atoms provided herein. Over five hundred conjugate parameters are calculated using a unique solution of the two-electron atom without any adjustable parameters to achieve overall agreement to the level obtainable considering the error in the measurements and the fundamental constants in the closed-form equations [5].

The background theory of classical quantum mechanics (CQM) for the physical solutions of atoms and atomic ions is disclosed in R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, January 2000 Edition, BlackLight Power, Inc., Cranbury, New Jersey, (" '00 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ, 08512; R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, September 2001 Edition, BlackLight Power, Inc., Cranbury, New Jersey, Distributed by Amazon.com (" '01 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ, 08512; R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, July 2004 Edition, BlackLight Power, Inc., Cranbury, New Jersey, (" '04 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ, 08512; R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, January 2005 Edition, BlackLight Power, Inc., Cranbury, New Jersey, (" '05 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ, 08512 (posted at www.blacklightpower.com and filed as a U.S. Provisional Application on January 3, 2005, entitled "The Grand Unified Theory of Classical Quantum Mechanics," attorney docket No. 62226-BOOK1); in prior PCT applications PCT/US02/35872; PCT/US02/06945; PCT/US02/06955; PCT/US01/09055; PCT/US01/ 25954; PCT/US00/20820; PCT/US00/20819; PCT/US00/09055; PCT/US99/17171; PCT/US99/17129; PCT/US 98/22822; PCT/US98/14029; PCT/US96/07949; PCT/US94/02219; PCT/US91/08496; PCT/US90/01998; and PCT/US89/05037 and U.S. Patent No. 6,024,935; the entire disclosures of which are all incorporated herein by reference; (hereinafter "Mills Prior Publications").

SUMMARY OF THE INVENTION

An object of the present invention is to solve the charge (mass) and current-density functions of atoms and atomic ions from first principles. In an embodiment, the solution is derived from Maxwell's equations invoking the constraint that the bound electron does not radiate even though it undergoes acceleration.

Another objective of the present invention is to generate a readout, display, image, or other output of the solutions so that the nature of atoms and atomic ions can be better understood and applied to predict reactivity and physical properties of atoms, ions and compounds.

Another objective of the present invention is to apply the methods and systems of solving the nature of bound electrons and its rendering to numerical or graphical form to all atoms and atomic ions.

These objectives and other objectives are met by a system of computing and rendering the nature of bound atomic and atomic ionic electrons from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration, comprising:

processing means for processing and solving the equations for charge, mass, and current density functions of electron(s) in a selected atom or ion, wherein the equations are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration; and

a display in communication with the processing means for displaying the current and charge density representation of the electron(s) of the selected atom or ion.

These objectives and other objectives are also met by a system of computing the nature of bound atomic and atomic ionic electrons from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration, comprising:

processing means for processing and solving the equations for charge, mass, and current density functions of electron(s) in selected atoms or ions, wherein the equations are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration; and

output means for outputting the solutions of the charge, mass, and current density functions of the atoms and atomic ions.

These objectives and other objectives are further met by a method comprising the steps of;

a.) inputting electron functions that are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration;

b.) inputting a trial electron configuration;

c.) inputting the corresponding centrifugal, Coulombic, diamagnetic and paramagnetic forces,

d.) forming the force balance equation comprising the centrifugal force equal to the sum of the Coulombic, diamagnetic and paramagnetic forces;

e.) solving the force balance equation for the electron radii;

f.) calculating the energy of the electrons using the radii and the corresponding electric and magnetic energies;

g.) repeating Steps a-f for all possible electron configurations, and

h.) outputting the lowest energy configuration and the corresponding electron radii for that configuration.

The invention will now be described with reference to classical quantum mechanics. A theory of classical quantum mechanics (CQM) was derived from first principles that successfully applies physical laws on all scales [2-6], and the mathematical connection with the Schrödinger equation to relate it to physical laws was discussed previously [27]. The physical approach based on Maxwell's equations was applied to multielectron atoms that were solved exactly. The classical predictions of the ionization energies were solved for the physical electrons comprising concentric orbitspheres ("bubble-like" charge-density functions) that are electrostatic and magnetostatic corresponding to a constant charge distribution and a constant current corresponding to spin angular momentum. Alternatively, the charge is a superposition of a constant and a dynamical component. In the latter case, charge density waves on the surface are time and spherically harmonic and correspond additionally to electron orbital angular momentum that superimposes the spin angular momentum. Thus, the electrons of multielectron atoms all exist as orbitspheres of discrete radii which are given by r_n of the radial Dirac delta function, $\delta(r-r_n)$. These electron orbitspheres may be spin paired or unpaired depending on the force balance which applies to each electron. Ultimately, the electron configuration must be a minimum of energy. Minimum energy configurations are given by solutions to Laplace's equation. As demonstrated previously, this general solution also gives the functions of the resonant photons of excited states [4]. It was found that electrons of an atom with the same principal and ℓ quantum numbers align parallel until each of the m_ℓ levels are occupied, and then pairing occurs until

each of the m_ℓ levels contain paired electrons. The electron configuration for one through twenty-electron atoms that achieves an energy minimum is: $1s < 2s < 2p < 3s < 3p < 4s$. In each case, the corresponding force balance of the central Coulombic, paramagnetic, and diamagnetic forces was derived for each n-electron atom that was solved for the radius of each electron. The central Coulombic force was that of a point charge at the origin since the electron charge-density functions are spherically symmetrical with a time dependence that was nonradiative. This feature eliminated the electron-electron repulsion terms and the intractable infinities of quantum mechanics and permitted general solutions. The ionization energies were obtained using the calculated radii in the determination of the Coulombic and any magnetic energies. The radii and ionization energies for all cases were given by equations having fundamental constants and each nuclear charge, Z, only. The predicted ionization energies and electron configurations given in TABLES I-XXIII are in remarkable agreement with the experimental values known for 400 atoms and ions.

The presented exact physical solutions for the atom and all ions having a given number of electrons can be used to predict the properties of elements and engineer compositions of matter in a manner which is not possible using quantum mechanics.

In an embodiment., the physical, Maxwellian solutions for the dimensions and energies of atom and atomic ions are processed with a processing means to produce an output. Embodiments of the system for performing computing and rendering of the nature of the bound atomic and atomic-ionic electrons using the physical solutions may comprise a general purpose computer. Such a general purpose computer may have any number of basic configurations. For example, such a general purpose computer may comprise a central processing unit (CPU), one or more specialized processors, system memory, a mass storage device such as a magnetic disk, an optical disk, or other storage device, an input means such as a keyboard or mouse, a display device, and a printer or other output device. A system implementing the present invention can also comprise a special purpose computer or other hardware system and all should be included within its scope.

BRIEF DESCRIPTION OF THE DRAWINGS

FIGURE 1 shows the orbitsphere in accordance with the present invention

that is a two dimensional spherical shell of zero thickness with the Bohr radius of the hydrogen atom, $r = a_H$.

FIGURE 2 shows the current pattern of the orbitsphere in accordance with the present invention from the perspective of looking along the z-axis. The current and charge density are confined to two dimensions at $r_n = nr_1$. The corresponding charge density function is uniform.

FIGURE 3 shows that the orbital function modulates the constant (spin) function (shown for t = 0; three-dimensional view).

FIGURE 4 shows the normalized radius as a function of the velocity due to relativistic contraction, and

FIGURE 5 shows the magnetic field of an electron orbitsphere (z-axis defined as the vertical axis).

DETAILED DESCRIPTION OF THE INVENTION

The following preferred embodiments of the invention disclose numerous calculations which are merely intended as illustrative examples. Based on the detailed written description, one skilled in the art would easily be able to practice this invention within other like calculations to produce the desired result without undue effort.

ONE-ELECTRON ATOMS

One-electron atoms include the hydrogen atom, He^+, Li^{2+}, Be^{3+}, and so on. The mass-energy and angular momentum of the electron are constant; this requires that the equation of motion of the electron be temporally and spatially harmonic. Thus, the classical wave equation applies and

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]\rho(r,\theta,\phi,t) = 0 \qquad (1)$$

where $\rho(r,\theta,\phi,t)$ is the time dependent charge density function of the electron in time and space. In general, the wave equation has an infinite number of solutions. To arrive at the solution which represents the electron, a suitable boundary condition must be imposed. It is well known from experiments that each single atomic electron of a given isotope radiates to the same stable state. Thus, the physical boundary condition of nonradiation of the bound electron was imposed on the solution of the wave equation for the time dependent charge density function of the

electron [2, 4]. The condition for radiation by a moving point charge given by Haus [16] is that its spacetime Fourier transform does possess components that are synchronous with waves traveling at the speed of light. Conversely, it is proposed that the condition for nonradiation by an ensemble of moving point charges that comprises a current density function is

For non-radiative states, the current-density function must NOT possess spacetime Fourier components that are synchronous with waves traveling at the speed of light.

The time, radial, and angular solutions of the wave equation are separable. The motion is time harmonic with frequency ω_n. A constant angular function is a solution to the wave equation. Solutions of the Schrödinger wave equation comprising a radial function radiate according to Maxwell's equation as shown previously by application of Haus' condition [4]. In fact, it was found that any function which permitted radial motion gave rise to radiation. A radial function which does satisfy the boundary condition is a radial delta function

$$f(r) = \frac{1}{r^2}\delta(r - r_n) \tag{2}$$

This function defines a constant charge density on a spherical shell where $r_n = nr_1$ wherein n is an integer in an excited state, and Eq. (1) becomes the two-dimensional wave equation plus time with separable time and angular functions. Given time harmonic motion and a radial delta function, the relationship between an allowed radius and the electron wavelength is given by

$$2\pi r_n = \lambda_n \tag{3}$$

where the integer subscript n here and in Eq. (2) is determined during photon absorption as given in the Excited States of the One-Electron Atom (Quantization) section of Ref. [4]. Using the observed de Broglie relationship for the electron mass where the coordinates are spherical,

$$\lambda_n = \frac{h}{p_n} = \frac{h}{m_e v_n} \tag{4}$$

and the magnitude of the velocity for *every* point on the orbitsphere is

$$v_n = \frac{\hbar}{m_e r_n} \tag{5}$$

The sum of the $|\mathbf{L}_i|$, the magnitude of the angular momentum of each infinitesimal point of the orbitsphere of mass m_i, must be constant. The constant is $\hbar$.

$$\sum|\mathbf{L}_i| = \sum|\mathbf{r} \times m_i\mathbf{v}| = m_e r_n \frac{\hbar}{m_e r_n} = \hbar \quad (6)$$

Thus, an electron is a spinning, two-dimensional spherical surface (zero thickness), called an *electron orbitsphere* shown in Figure 1, that can exist in a bound state at only specified distances from the nucleus determined by an energy minimum. The corresponding current function shown in Figure 2 which gives rise to the phenomenon of *spin* is derived in the Spin Function section. (See the Orbitsphere Equation of Motion for $\ell = 0$ of Ref. [4] at Chp. 1.)

Nonconstant functions are also solutions for the angular functions. To be a harmonic solution of the wave equation in spherical coordinates, these angular functions must be spherical harmonic functions [18]. A zero of the spacetime Fourier transform of the product function of two spherical harmonic angular functions, a time harmonic function, and an unknown radial function is sought. The solution for the radial function which satisfies the boundary condition is also a delta function given by Eq. (2). Thus, bound electrons are described by a charge-density (mass-density) function which is the product of a radial delta function, two angular functions (spherical harmonic functions), and a time harmonic function.

$$\rho(r,\theta,\phi,t) = f(r)A(\theta,\phi,t) = \frac{1}{r^2}\delta(r-r_n)A(\theta,\phi,t); \qquad A(\theta,\phi,t) = Y(\theta,\phi)k(t) \quad (7)$$

In these cases, the spherical harmonic functions correspond to a traveling charge density wave confined to the spherical shell which gives rise to the phenomenon of orbital angular momentum. The orbital functions which modulate the constant "spin" function shown graphically in Figure 3 are given in the Angular Functions section.

SPIN FUNCTION

The orbitsphere spin function comprises a constant charge (current) density function with moving charge confined to a two-dimensional spherical shell. The magnetostatic current pattern of the orbitsphere spin function comprises an infinite series of correlated orthogonal great circle current loops wherein each point charge (current) density element moves time harmonically with constant angular velocity

$$\omega_n = \frac{\hbar}{m_e r_n^2} \quad (8)$$

The uniform current density function $Y_0^0(\phi,\theta)$, the orbitsphere equation of motion of the electron (Eqs. (13-14)), corresponding to the constant charge function of the orbitsphere that gives rise to the spin of the electron is generated from a basis set current-vector field defined as the orbitsphere current-vector field ("orbitsphere-cvf"). This in turn is generated over the surface by two complementary steps of an infinite series of nested rotations of two orthogonal great circle current loops where the coordinate axes rotate with the two orthogonal great circles that serve as a basis set. The algorithm to generate the current density function rotates the great circles and the corresponding x'y'z' coordinates relative to the xyz frame. Each infinitesimal rotation of the infinite series is about the new i'-axis and new j'-axis which results from the preceding such rotation. Each element of the current density function is obtained with each conjugate set of rotations. In Appendix III of Ref. [4], the *continuous* uniform electron current density function $Y_0^0(\phi,\theta)$ having the same angular momentum components as that of the orbitsphere-cvf is then exactly generated from this orbitsphere-cvf as a basis element by a convolution operator comprising an autocorrelation-type function.

For Step One, the current density elements move counter clockwise on the great circle in the y'z'-plane and move clockwise on the great circle in the x'z'-plane. The great circles are rotated by an infinitesimal angle $\pm\Delta\alpha_{i'}$ (a positive rotation around the x'-axis or a negative rotation about the z'-axis for Steps One and Two, respectively) and then by $\pm\Delta\alpha_{j'}$ (a positive rotation around the new y'-axis or a positive rotation about the new x'-axis for Steps One and Two, respectively). The coordinates of each point on each rotated great circle (x',y',z') is expressed in terms of the first (x,y,z) coordinates by the following transforms where clockwise rotations and motions are defined as positive looking along the corresponding axis:

Step One

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(\Delta\alpha_y) & 0 & -\sin(\Delta\alpha_y) \\ 0 & 1 & 0 \\ \sin(\Delta\alpha_y) & 0 & \cos(\Delta\alpha_y) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Delta\alpha_x) & \sin(\Delta\alpha_x) \\ 0 & -\sin(\Delta\alpha_x) & \cos(\Delta\alpha_x) \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(\Delta\alpha_y) & \sin(\Delta\alpha_y)\sin(\Delta\alpha_x) & -\sin(\Delta\alpha_y)\cos(\Delta\alpha_x) \\ 0 & \cos(\Delta\alpha_x) & \sin(\Delta\alpha_x) \\ \sin(\Delta\alpha_y) & -\cos(\Delta\alpha_y)\sin(\Delta\alpha_x) & \cos(\Delta\alpha_y)\cos(\Delta\alpha_x) \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad (9)$$

Step Two

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Delta\alpha_x) & \sin(\Delta\alpha_x) \\ 0 & -\sin(\Delta\alpha_x) & \cos(\Delta\alpha_x) \end{bmatrix} \begin{bmatrix} \cos(\Delta\alpha_z) & \sin(\Delta\alpha_z) & 0 \\ -\sin(\Delta\alpha_z) & \cos(\Delta\alpha_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(\Delta\alpha_z) & \sin(\Delta\alpha_z) & 0 \\ -\cos(\Delta\alpha_x)\sin(\Delta\alpha_z) & \cos(\Delta\alpha_x)\cos(\Delta\alpha_z) & \sin(\Delta\alpha_x) \\ \sin(\Delta\alpha_x)\sin(\Delta\alpha_z) & -\sin(\Delta\alpha_x)\cos(\Delta\alpha_z) & \cos(\Delta\alpha_x) \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} \quad (10)$$

where the angular sum is $\lim_{\Delta\alpha\to 0} \sum_{n=1}^{\frac{\frac{\sqrt{2}}{2}\pi}{|\Delta\alpha_{i',j'}|}} |\Delta\alpha_{i',j'}| = \frac{\sqrt{2}}{2}\pi$.

The orbitsphere-cvf is given by n reiterations of Eqs. (9) and (10) for each point on each of the two orthogonal great circles during each of Steps One and Two. The output given by the non-primed coordinates is the input of the next iteration corresponding to each successive nested rotation by the infinitesimal angle $\pm\Delta\alpha_{i'}$ or $\pm\Delta\alpha_{j'}$ where the magnitude of the angular sum of the n rotations about each of the i'-axis and the j'-axis is $\frac{\sqrt{2}}{2}\pi$. Half of the orbitsphere-cvf is generated during each of Steps One and Two.

Following Step Two, in order to match the boundary condition that the magnitude of the velocity at any given point on the surface is given by Eq. (5), the output half of the orbitsphere-cvf is rotated clockwise by an angle of $\frac{\pi}{4}$ about the z-axis. Using Eq. (10) with $\Delta\alpha_{z'} = \frac{\pi}{4}$ and $\Delta\alpha_{x'} = 0$ gives the rotation. Then, the one half of the orbitsphere-cvf generated from Step One is superimposed with the complementary half obtained from Step Two following its rotation about the z-axis of $\frac{\pi}{4}$ to give the basis function to generate $Y_0^0(\phi,\theta)$, the orbitsphere equation of motion of the electron.

The current pattern of the orbitsphere-cvf generated by the nested rotations of the orthogonal great circle current loops is a continuous and total coverage of the spherical surface, but it is shown as a visual representation using 6 degree increments of the infinitesimal angular variable $\pm\Delta\alpha_{i'}$ and $\pm\Delta\alpha_{j'}$ of Eqs. (9) and (10) from the perspective of the z-axis in Figure 2. In each case, the complete orbitsphere-cvf current pattern corresponds all the orthogonal-great-circle elements which are generated by the rotation of the basis-set according to Eqs. (9) and (10) where $\pm\Delta\alpha_{i'}$ and $\pm\Delta\alpha_{j'}$ approach zero and the summation of the infinitesimal angular rotations of $\pm\Delta\alpha_{i'}$ and $\pm\Delta\alpha_{j'}$ about the successive i'-axes and j'-axes is $\frac{\sqrt{2}}{2}\pi$ for each Step. The current pattern gives rise to the phenomenon corresponding to the spin quantum number. The details of the derivation of the spin function are given in Ref. [2] and Chp. 1 of Ref. [4].

The resultant angular momentum projections of $\mathbf{L}_{xy} = \frac{\hbar}{4}$ and $\mathbf{L}_z = \frac{\hbar}{2}$ meet the boundary condition for the unique current having an angular velocity magnitude at each point on the surface given by Eq. (5) and give rise to the Stern Gerlach experiment as shown in Ref. [4]. The further constraint that the current density is uniform such that the charge density is uniform, corresponding to an equipotential, minimum energy surface is satisfied by using the orbitsphere-cvf as a basis element to generate $Y_0^0(\phi,\theta)$ using a convolution operator comprising an autocorrelation-type function as given in Appendix III of Ref. [4]. The operator comprises the convolution of each great circle current loop of the orbitsphere-cvf designated as the primary orbitsphere-cvf with a second orbitsphere-cvf designated as the secondary

orbitsphere-cvf wherein the convolved secondary elements are matched for orientation, angular momentum, and phase to those of the primary. The resulting exact uniform current distribution obtained from the convolution has the same angular momentum distribution, resultant, $\mathbf{L}_R$, and components of $\mathbf{L}_{xy} = \frac{\hbar}{4}$ and $\mathbf{L}_z = \frac{\hbar}{2}$ as those of the orbitsphere-cvf used as a primary basis element.

ANGULAR FUNCTIONS

The time, radial, and angular solutions of the wave equation are separable. Also based on the radial solution, the angular charge and current-density functions of the electron, $A(\theta,\phi,t)$, must be a solution of the wave equation in two dimensions (plus time),

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]A(\theta,\phi,t) = 0 \qquad (11)$$

where $\rho(r,\theta,\phi,t) = f(r)A(\theta,\phi,t) = \frac{1}{r^2}\delta(r-r_n)A(\theta,\phi,t)$ and $A(\theta,\phi,t) = Y(\theta,\phi)k(t)$

$$\left[\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)_{r,\phi} + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)_{r,\theta} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]A(\theta,\phi,t) = 0 \qquad (12)$$

where v is the linear velocity of the electron. The charge-density functions including the time-function factor are

$\ell = 0$

$$\rho(r,\theta,\phi,t) = \frac{e}{8\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi) + Y_\ell^m(\theta,\phi)\right] \qquad (13)$$

$\ell \neq 0$

$$\rho(r,\theta,\phi,t) = \frac{e}{4\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi) + \mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\}\right] \qquad (14)$$

where $Y_\ell^m(\theta,\phi)$ are the spherical harmonic functions that spin about the z-axis with angular frequency ω_n with $Y_0^0(\theta,\phi)$ the constant function.

$\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\} = P_\ell^m(\cos\theta)\cos\left(m\phi + \omega_n' t\right)$ where to keep the form of the spherical harmonic as a traveling wave about the z-axis, $\omega_n' = m\omega_n$.

ACCELERATION WITHOUT RADIATION

Special Relativistic Correction to the Electron Radius

The relationship between the electron wavelength and its radius is given by Eq. (3) where λ is the de Broglie wavelength. For each current density element of the spin function, the distance along each great circle in the direction of instantaneous motion undergoes length contraction and time dilation. Using a phase matching condition, the wavelengths of the electron and laboratory inertial frames are equated, and the corrected radius is given by

$$r_n = r_n'\left[\sqrt{1-\left(\frac{v}{c}\right)^2}\,\sin\left[\frac{\pi}{2}\left(1-\left(\frac{v}{c}\right)^2\right)^{3/2}\right]+\frac{1}{2\pi}\cos\left[\frac{\pi}{2}\left(1-\left(\frac{v}{c}\right)^2\right)^{3/2}\right]\right] \qquad (15)$$

where the electron velocity is given by Eq. (5). (See Ref. [4] Chp. 1, Special Relativistic Correction to the Ionization Energies section). $\frac{e}{m_e}$ of the electron, the electron angular momentum of $\hbar$, and μ_B are invariant, but the mass and charge densities increase in the laboratory frame due to the relativistically contracted electron radius. As $v \to c$, $r/r' \to \frac{1}{2\pi}$ and $r = \lambda$ as shown in Figure 4.

Nonradiation Based on the Spacetime Fourier Transform of the Electron Current

Although an accelerated *point* particle radiates, an *extended distribution* modeled as a superposition of accelerating charges does not have to radiate [14, 16, 19-21]. The Fourier transform of the electron charge density function given by Eq. (7) is a solution of the three-dimensional wave equation in frequency space ($\mathbf{k}, \omega\, space$) as given in Chp 1, Spacetime Fourier Transform of the Electron Function section, of Ref. [4]. Then the corresponding Fourier transform of the current density function $K(s,\Theta,\Phi,\omega)$ is given by multiplying by the constant angular frequency.

$$K(s,\Theta,\Phi,\omega) = 4\pi\omega_n \frac{\sin(2s_n r_n)}{2s_n r_n}$$

$$\otimes 2\pi \sum_{\upsilon=1}^{\infty} \frac{(-1)^{\upsilon-1}(\pi\sin\Theta)^{2(\upsilon-1)}}{(\upsilon-1)!(\upsilon-1)!} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\upsilon+\frac{1}{2}\right)}{(\pi\cos\Theta)^{2\upsilon+1}2^{\upsilon+1}} \frac{2\upsilon!}{(\upsilon-1)!} s^{-2\upsilon}$$

$$\otimes 2\pi \sum_{\upsilon=1}^{\infty} \frac{(-1)^{\upsilon-1}(\pi\sin\Phi)^{2(\upsilon-1)}}{(\upsilon-1)!(\upsilon-1)!} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\upsilon+\frac{1}{2}\right)}{(\pi\cos\Phi)^{2\upsilon+1}2^{\upsilon+1}} \frac{2\upsilon!}{(\upsilon-1)!} s^{-2\upsilon} \qquad (16)$$

$$\frac{1}{4\pi}[\delta(\omega-\omega_n)+\delta(\omega+\omega_n)]$$

$\mathbf{s}_n \bullet \mathbf{v}_n = \mathbf{s}_n \bullet \mathbf{c} = \omega_n$ implies $r_n = \lambda_n$ which is given by Eq. (15) in the case that k is the lightlike k^0. In this case, Eq. (16) vanishes. Consequently, spacetime harmonics of $\frac{\omega_n}{c} = k$ or $\frac{\omega_n}{c}\sqrt{\frac{\varepsilon}{\varepsilon_o}} = k$ for which the Fourier transform of the current-density function is nonzero do not exist. Radiation due to charge motion does not occur in any medium when this boundary condition is met. Nonradiation is also determined from the fields based on Maxwell's equations as given in the Nonradiation Based on the Electromagnetic Fields and the Poynting Power Vector section *infra.*

Nonradiation Based on the Electron Electromagnetic Fields and the Poynting Power Vector

A point charge undergoing periodic motion accelerates and as a consequence radiates according to the Larmor formula:

$$P = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{3c^3}a^2 \qquad (17)$$

where e is the charge, a is its acceleration, ε_0 is the permittivity of free space, and c is the speed of light. Although an accelerated *point* particle radiates, an *extended distribution* modeled as a superposition of accelerating charges does not have to radiate [14, 16, 19-21]. In Ref. [2] and Appendix I, Chp. 1 of Ref. [4], the electromagnetic far field is determined from the current distribution in order to obtain the condition, if it exists, that the electron current distribution must satisfy such that the electron does not radiate. The current follows from Eqs. (13-14). The currents corresponding to Eq. (13) and first term of Eq. (14) are static. Thus, they are trivially nonradiative. The current due to the time dependent term of Eq. (14) corresponding to p, d, f, etc. orbitals is

$$\mathbf{J} = \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2} N[\delta(r-r_n)]\mathrm{Re}\{Y_\ell^m(\theta,\phi)\}[\mathbf{u}(t)\times\mathbf{r}]$$
$$= \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2} N'[\delta(r-r_n)]\left(P_\ell^m(\cos\theta)\cos\left(m\phi+\omega_n' t\right)\right)[\mathbf{u}\times\mathbf{r}] \quad (18)$$
$$= \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2} N'[\delta(r-r_n)]\left(P_\ell^m(\cos\theta)\cos\left(m\phi+\omega_n' t\right)\right)\sin\theta\hat{\phi}$$

where to keep the form of the spherical harmonic as a traveling wave about the z-axis, $\omega_n' = m\omega_n$ and N and N' are normalization constants. The vectors are defined as

$$\hat{\phi} = \frac{\hat{u}\times\hat{r}}{|\hat{u}\times\hat{r}|} = \frac{\hat{u}\times\hat{r}}{\sin\theta}; \qquad \hat{u} = \hat{z} = orbital\ axis \quad (19)$$

$$\hat{\theta} = \hat{\phi}\times\hat{r} \quad (20)$$

"^" denotes the unit vectors $\hat{u} \equiv \frac{\mathbf{u}}{|\mathbf{u}|}$, non-unit vectors are designed in bold, and the current function is normalized. For the electron source current given by Eq. (18), each comprising a multipole of order (ℓ,m) with a time dependence $e^{i\omega_n t}$, the far-field solutions to Maxwell's equations are given by

$$\mathbf{B} = -\frac{i}{k}a_M(\ell,m)\nabla\times g_\ell(kr)\mathbf{X}_{\ell.m}$$
$$\mathbf{E} = a_M(\ell,m)g_\ell(kr)\mathbf{X}_{\ell.m} \quad (21)$$

and the time-averaged power radiated per solid angle $\frac{dP(\ell,m)}{d\Omega}$ is

$$\frac{dP(\ell,m)}{d\Omega} = \frac{c}{8\pi k^2}|a_M(\ell,m)|^2|\mathbf{X}_{\ell.m}|^2 \quad (22)$$

where $a_M(\ell,m)$ is

$$a_M(\ell,m) = \frac{-ek^2}{c\sqrt{\ell(\ell+1)}}\frac{\omega_n}{2\pi}Nj_\ell(kr_n)\Theta\sin(mks) \quad (23)$$

In the case that k is the lightlike k^0, then $k = \omega_n/c$, in Eq. (23), and Eqs. (21-22) vanishes for

$$s = vT_n = R = r_n = \lambda_n \quad (24)$$

There is no radiation.

MAGNETIC FIELD EQUATIONS OF THE ELECTRON

The orbitsphere is a shell of negative charge current comprising correlated charge motion along great circles. For ℓ = 0, the orbitsphere gives rise to a magnetic moment of 1 Bohr magneton [22]. (The details of the derivation of the magnetic parameters including the electron g factor are given in Ref. [2] and Chp. 1 of Ref. [4].)

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274\,X\,10^{-24}\,JT^{-1} \quad (25)$$

The magnetic field of the electron shown in Figure 5 is given by

$$\mathbf{H} = \frac{e\hbar}{m_e r_n^3}(\mathbf{i}_r \cos\theta - \mathbf{i}_\theta \sin\theta) \qquad \text{for } r < r_n \quad (26)$$

$$\mathbf{H} = \frac{e\hbar}{2m_e r^3}(\mathbf{i}_r 2\cos\theta + \mathbf{i}_\theta \sin\theta) \qquad \text{for } r > r_n \quad (27)$$

The energy stored in the magnetic field of the electron is

$$E_{mag} = \frac{1}{2}\mu_o \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} H^2 r^2 \sin\theta dr d\theta d\Phi \quad (28)$$

$$E_{mag\,\text{total}} = \frac{\pi\mu_o e^2\hbar^2}{m_e^2 r_1^3} \quad (29)$$

STERN-GERLACH EXPERIMENT

The Stern-Gerlach experiment implies a magnetic moment of one Bohr magneton and an associated angular momentum quantum number of 1/2. Historically, this quantum number is called the spin quantum number, s ($s = \frac{1}{2}$; $m_s = \pm\frac{1}{2}$). The superposition of the vector projection of the orbitsphere angular momentum on the z-axis is $\frac{\hbar}{2}$ with an orthogonal component of $\frac{\hbar}{4}$. Excitation of a resonant Larmor precession gives rise to $\hbar$ on an axis $\mathbf{S}$ that precesses about the z-axis called the spin axis at the Larmor frequency at an angle of $\theta = \frac{\pi}{3}$ to give a perpendicular projection of

$$\mathbf{S}_\perp = \pm\sqrt{\frac{3}{4}}\hbar \quad (30)$$

and a projection onto the axis of the applied magnetic field of

$$\mathbf{S}_\parallel = \pm\frac{\hbar}{2} \quad (31)$$

The superposition of the $\frac{\hbar}{2}$, z-axis component of the orbitsphere angular momentum and the $\frac{\hbar}{2}$, z-axis component of $\mathbf{S}$ gives $\hbar$ corresponding to the observed electron magnetic moment of a Bohr magneton, μ_B.

ELECTRON g FACTOR

Conservation of angular momentum of the orbitsphere permits a discrete change of its "kinetic angular momentum" ($\mathbf{r} \times m\mathbf{v}$) by the applied magnetic field of $\frac{\hbar}{2}$, and concomitantly the "potential angular momentum" ($\mathbf{r} \times e\mathbf{A}$) must change by

$-\frac{\hbar}{2}$.

$$\Delta \mathbf{L} = \frac{\hbar}{2} - \mathbf{r} \times e\mathbf{A} \tag{32}$$

$$= \left[\frac{\hbar}{2} - \frac{e\phi}{2\pi}\right]\hat{z} \tag{33}$$

In order that the change of angular momentum, $\Delta \mathbf{L}$, equals zero, ϕ must be $\Phi_0 = \frac{h}{2e}$, the magnetic flux quantum. The magnetic moment of the electron is parallel or antiparallel to the applied field only. During the spin-flip transition, power must be conserved. Power flow is governed by the Poynting power theorem,

$$\nabla \bullet (\mathbf{E} \times \mathbf{H}) = -\frac{\partial}{\partial t}\left[\frac{1}{2}\mu_o \mathbf{H} \bullet \mathbf{H}\right] - \frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon_o \mathbf{E} \bullet \mathbf{E}\right] - \mathbf{J} \bullet \mathbf{E} \tag{34}$$

Eq. (35) gives the total energy of the flip transition which is the sum of the energy of reorientation of the magnetic moment (1st term), the magnetic energy (2nd term), the electric energy (3rd term), and the dissipated energy of a fluxon treading the orbitsphere (4th term), respectively,

$$\Delta E_{mag}^{spin} = 2\left(1 + \frac{\alpha}{2\pi} + \frac{2}{3}\alpha^2\left(\frac{\alpha}{2\pi}\right) - \frac{4}{3}\left(\frac{\alpha}{2\pi}\right)^2\right)\mu_B B \tag{35}$$

$$\Delta E_{mag}^{spin} = g\mu_B B \tag{36}$$

where the stored magnetic energy corresponding to the $\frac{\partial}{\partial t}\left[\frac{1}{2}\mu_o \mathbf{H} \bullet \mathbf{H}\right]$ term increases, the stored electric energy corresponding to the $\frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon_o \mathbf{E} \bullet \mathbf{E}\right]$ term increases, and the $\mathbf{J} \bullet \mathbf{E}$ term is dissipative. The spin-flip transition can be considered as involving a magnetic moment of g times that of a Bohr magneton. The g factor is redesignated the fluxon g factor as opposed to the anomalous g factor. Using $\alpha^{-1} = 137.03603(82)$, the calculated value of $\frac{g}{2}$ is $1.001\ 159\ 652\ 137$. The experimental value [23] of $\frac{g}{2}$ is $1.001\ 159\ 652\ 188(4)$.

SPIN AND ORBITAL PARAMETERS

The total function that describes the spinning motion of each electron orbitsphere is composed of two functions. One function, the spin function, is spatially uniform over the orbitsphere, spins with a quantized angular velocity, and gives rise to spin angular momentum. The other function, the modulation function, can be spatially uniform—in which case there is no orbital angular momentum and the magnetic moment of the electron orbitsphere is one Bohr magneton—or not spatially uniform—in which case there is orbital angular momentum. The modulation

function also rotates with a quantized angular velocity.

The spin function of the electron corresponds to the nonradiative $n = 1$, $\ell = 0$ state of atomic hydrogen which is well known as an s state or orbital. (See Figure 1 for the charge function and Figure 2 for the current function.) In cases of orbitals of heavier elements and excited states of one electron atoms and atoms or ions of heavier elements with the ℓ quantum number not equal to zero and which are not constant as given by Eq. (13), the constant spin function is modulated by a time and spherical harmonic function as given by Eq. (14) and shown in Figure 3. The modulation or traveling charge density wave corresponds to an orbital angular momentum in addition to a spin angular momentum. These states are typically referred to as p, d, f, etc. orbitals. Application of Haus's [16] condition also predicts nonradiation for a constant spin function modulated by a time and spherically harmonic orbital function. There is acceleration without radiation as also shown in the Nonradiation Based on the Electron Electromagnetic Fields and the Poynting Power Vector section. (Also see Pearle, Abbott and Griffiths, Goedecke, and Daboul and Jensen [14, 19-21]). However, in the case that such a state arises as an excited state by photon absorption, it is radiative due to a radial dipole term in its current density function since it possesses spacetime Fourier Transform components synchronous with waves traveling at the speed of light [16]. (See Instability of Excited States section of Ref. [4].)

Moment of Inertia and Spin and Rotational Energies

The moments of inertia and the rotational energies as a function of the ℓ quantum number for the solutions of the time-dependent electron charge density functions (Eqs. (13-14)) given in the Angular Functions section are solved using the rigid rotor equation [24]. The details of the derivations of the results as well as the demonstration that Eqs. (13-14) with the results given *infra.* are solutions of the wave equation are given in Chp 1, Rotational Parameters of the Electron (Angular Momentum, Rotational Energy, Moment of Inertia) section, of Ref. [4].

$\ell = 0$

$$I_z = I_{spin} = \frac{m_e r_n^2}{2} \tag{37}$$

$$L_z = I\omega \mathbf{i}_z = \pm\frac{\hbar}{2} \tag{38}$$

$$E_{rotational} = E_{rotational,\,spin} = \frac{1}{2}\left[I_{spin}\left(\frac{\hbar}{m_e r_n^{\,2}}\right)^2\right] = \frac{1}{2}\left[\frac{m_e r_n^{\,2}}{2}\left(\frac{\hbar}{m_e r_n^{\,2}}\right)^2\right] = \frac{1}{4}\left[\frac{\hbar^2}{2I_{spin}}\right] \quad (39)$$

$\ell ? 0$

$$I_{orbital} = m_e r_n^{\,2}\left[\frac{\ell(\ell+1)}{\ell^2+\ell+1}\right]^{\frac{1}{2}} \quad (40)$$

$$L_z = m\hbar \quad (41)$$

$$L_{z\,total} = L_{z\,spin} + L_{z\,orbital} \quad (42)$$

$$E_{rotational,\,orbital} = \frac{\hbar^2}{2I}\left[\frac{\ell(\ell+1)}{\ell^2+2\ell+1}\right] \quad (43)$$

$$T = \frac{\hbar^2}{2m_e r_n^2} \quad (44)$$

$$\left\langle E_{rotational,\,orbital}\right\rangle = 0 \quad (45)$$

From Eq. (45), the time average rotational energy is zero; thus, the principal levels are degenerate except when a magnetic field is applied.

FORCE BALANCE EQUATION

The radius of the nonradiative ($n = 1$) state is solved using the electromagnetic force equations of Maxwell relating the charge and mass density functions wherein the angular momentum of the electron is given by Planck's constant bar [4]. The reduced mass arises naturally from an electrodynamic interaction between the electron and the proton of mass m_p.

$$\frac{m_e}{4\pi r_1^2}\frac{v_1^2}{r_1} = \frac{e}{4\pi r_1^2}\frac{Ze}{4\pi\varepsilon_o r_1^2} - \frac{1}{4\pi r_1^2}\frac{\hbar^2}{m_p r_n^3} \quad (46)$$

$$r_1 = \frac{a_H}{Z} \quad (47)$$

where a_H is the radius of the hydrogen atom.

ENERGY CALCULATIONS

From Maxwell's equations, the potential energy V, kinetic energy T, electric energy or binding energy E_{ele} are

$$V = \frac{-Ze^2}{4\pi\varepsilon_o r_1} = \frac{-Z^2e^2}{4\pi\varepsilon_o a_H} = -Z^2\,X\,4.3675\,X10^{-18}\,J = -Z^2\,X\,27.2\;eV \quad (48)$$

$$T = \frac{Z^2e^2}{8\pi\varepsilon_o a_H} = Z^2\,X\,13.59\;eV \quad (49)$$

$$T = E_{ele} = -\frac{1}{2}\varepsilon_o\int_{\infty}^{r_1}\mathbf{E}^2 dv \quad \text{where } \mathbf{E} = -\frac{Ze}{4\pi\varepsilon_o r^2} \quad (50)$$

$$E_{ele} = -\frac{Z^2 e^2}{8\pi\varepsilon_o a_H} = -Z^2 X 2.1786 X 10^{-18} J = -Z^2 X 13.598 eV \quad (51)$$

The calculated Rydberg constant is $10,967,758\ m^{-1}$; the experimental Rydberg constant is $10,967,758\ m^{-1}$. For increasing Z, the velocity becomes a significant fraction of the speed of light; thus, special relativistic corrections were included in the calculation of the ionization energies of one-electron atoms that are given in TABLE I.

TABLE I. Relativistically corrected ionization energies for some one-electron atoms.

One e Atom	Z	γ^* a	Theoretical Ionization Energies (eV) b	Experimental Ionization Energies (eV) c	Relative Difference between Experimental and Calculated d
H	1	1.000007	13.59838	13.59844	0.00000
He^{+}	2	1.000027	54.40941	54.41778	0.00015
Li^{2+}	3	1.000061	122.43642	122.45429	0.00015
Be^{3+}	4	1.000109	217.68510	217.71865	0.00015
B^{4+}	5	1.000172	340.16367	340.2258	0.00018
C^{5+}	6	1.000251	489.88324	489.99334	0.00022
N^{6+}	7	1.000347	666.85813	667.046	0.00028
O^{7+}	8	1.000461	871.10635	871.4101	0.00035
F^{8+}	9	1.000595	1102.65013	1103.1176	0.00042
Ne^{9+}	10	1.000751	1361.51654	1362.1995	0.00050
Na^{10+}	11	1.000930	1647.73821	1648.702	0.00058
Mg^{11+}	12	1.001135	1961.35405	1962.665	0.00067
Al^{12+}	13	1.001368	2302.41017	2304.141	0.00075
Si^{13+}	14	1.001631	2670.96078	2673.182	0.00083
P^{14+}	15	1.001927	3067.06918	3069.842	0.00090
S^{15+}	16	1.002260	3490.80890	3494.1892	0.00097
Cl^{16+}	17	1.002631	3942.26481	3946.296	0.00102
Ar^{17+}	18	1.003045	4421.53438	4426.2296	0.00106
K^{18+}	19	1.003505	4928.72898	4934.046	0.00108
Ca^{19+}	20	1.004014	5463.97524	5469.864	0.00108
Sc^{20+}	21	1.004577	6027.41657	6033.712	0.00104
Ti^{21+}	22	1.005197	6619.21462	6625.82	0.00100
V^{22+}	23	1.005879	7239.55091	7246.12	0.00091
Cr^{23+}	24	1.006626	7888.62855	7894.81	0.00078
Mn^{24+}	25	1.007444	8566.67392	8571.94	0.00061
Fe^{25+}	26	1.008338	9273.93857	9277.69	0.00040
Co^{26+}	27	1.009311	10010.70111	10012.12	0.00014
Ni^{27+}	28	1.010370	10777.26918	10775.4	-0.00017
Cu^{28+}	29	1.011520	11573.98161	11567.617	-0.00055

a Eq. (1.250) (follows Eqs. (5), (15), and (47)).

b Eq. (1.251) (Eq. (51) times γ^*).

c From theoretical calculations, interpolation of H isoelectronic and Rydberg series, and experimental data [24-25].

d (Experimental-theoretical)/experimental.

TWO ELECTRON ATOMS

Two electron atoms may be solved from a central force balance equation with the nonradiation condition [4]. The force balance equation is

$$\frac{m_e}{4\pi r_2^2}\frac{v_2^2}{r_2} = \frac{e}{4\pi r_2^2}\frac{(Z-1)e}{4\pi\varepsilon_o r_2^2} + \frac{1}{4\pi r_2^2}\frac{\hbar^2}{Zm_e r_2^3}\sqrt{s(s+1)} \tag{52}$$

which gives the radius of both electrons as

$$r_2 = r_1 = a_0\left(\frac{1}{Z-1} - \frac{\sqrt{s(s+1)}}{Z(Z-1)}\right);\ s = \frac{1}{2} \tag{53}$$

IONIZATION ENERGIES CALCULATED USING THE POYNTING POWER THEOREM

For helium, which has no electric field beyond r_1

$$Ionization\ Energy(He) = -E(electric) + E(magnetic) \tag{54}$$

where,

$$E(electric) = -\frac{(Z-1)e^2}{8\pi\varepsilon_o r_1} \tag{55}$$

$$E(magnetic) = \frac{2\pi\mu_0 e^2\hbar^2}{m_e^2 r_1^3} \tag{56}$$

For $3 \le Z$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z}Magnetic\ Energy \tag{57}$$

For increasing Z, the velocity becomes a significant fraction of the speed of light; thus, special relativistic corrections were included in the calculation of the ionization energies of two-electron atoms that are given in TABLE II.

TABLE II. Relativistically corrected ionization energies for some two-electron atoms.

2 e Atom	Z	r_1 (a_o) [a]	Electric Energy [b] (eV)	Magnetic Energy [c] (eV)
He	2	0.566987	23.996467	0.590536
Li^{+}	3	0.35566	76.509	2.543
Be^{2+}	4	0.26116	156.289	6.423
B^{3+}	5	0.20670	263.295	12.956
C^{4+}	6	0.17113	397.519	22.828
N^{5+}	7	0.14605	558.958	36.728
O^{6+}	8	0.12739	747.610	55.340
F^{7+}	9	0.11297	963.475	79.352
Ne^{8+}	10	0.10149	1206.551	109.451
Na^{9+}	11	0.09213	1476.840	146.322
Mg^{10+}	12	0.08435	1774.341	190.652
Al^{11+}	13	0.07778	2099.05	243.13
Si^{12+}	14	0.07216	2450.98	304.44
P^{13+}	15	0.06730	2830.11	375.26
S^{14+}	16	0.06306	3236.46	456.30
Cl^{15+}	17	0.05932	3670.02	548.22
Ar^{16+}	18	0.05599	4130.79	651.72
K^{17+}	19	0.05302	4618.77	767.49
Ca^{18+}	20	0.05035	5133.96	896.20
Sc^{19+}	21	0.04794	5676.37	1038.56
Ti^{20+}	22	0.04574	6245.98	1195.24
V^{21+}	23	0.04374	6842.81	1366.92
Cr^{22+}	24	0.04191	7466.85	1554.31
Mn^{23+}	25	0.04022	8118.10	1758.08
Fe^{24+}	26	0.03867	8796.56	1978.92
Co^{25+}	27	0.03723	9502.23	2217.51
Ni^{26+}	28	0.03589	10235.12	2474.55
Cu^{27+}	29	0.03465	10995.21	2750.72

2 e Atom	Z	Velocity (m/s) [d]	γ^* [e]	Theoretical Ionization Energies [f] (eV)	Experimental Ionization Energies [g] (eV)	Relative Error [h]
He	2	3.85845E+06	1.000021	24.58750	24.58741	-0.000004
Li^{+}	3	6.15103E+06	1.00005	75.665	75.64018	-0.0003
Be^{2+}	4	8.37668E+06	1.00010	154.699	153.89661	-0.0052
B^{3+}	5	1.05840E+07	1.00016	260.746	259.37521	-0.0053
C^{4+}	6	1.27836E+07	1.00024	393.809	392.087	-0.0044
N^{5+}	7	1.49794E+07	1.00033	553.896	552.0718	-0.0033
O^{6+}	8	1.71729E+07	1.00044	741.023	739.29	-0.0023
F^{7+}	9	1.93649E+07	1.00057	955.211	953.9112	-0.0014
Ne^{8+}	10	2.15560E+07	1.00073	1196.483	1195.8286	-0.0005
Na^{9+}	11	2.37465E+07	1.00090	1464.871	1465.121	0.0002
Mg^{10+}	12	2.59364E+07	1.00110	1760.411	1761.805	0.0008
Al^{11+}	13	2.81260E+07	1.00133	2083.15	2085.98	0.0014
Si^{12+}	14	3.03153E+07	1.00159	2433.13	2437.63	0.0018
P^{13+}	15	3.25043E+07	1.00188	2810.42	2816.91	0.0023
S^{14+}	16	3.46932E+07	1.00221	3215.09	3223.78	0.0027
Cl^{15+}	17	3.68819E+07	1.00258	3647.22	3658.521	0.0031
Ar^{16+}	18	3.90705E+07	1.00298	4106.91	4120.8857	0.0034
K^{17+}	19	4.12590E+07	1.00344	4594.25	4610.8	0.0036
Ca^{18+}	20	4.34475E+07	1.00394	5109.38	5128.8	0.0038
Sc^{19+}	21	4.56358E+07	1.00450	5652.43	5674.8	0.0039
Ti^{20+}	22	4.78241E+07	1.00511	6223.55	6249	0.0041
V^{21+}	23	5.00123E+07	1.00578	6822.93	6851.3	0.0041
Cr^{22+}	24	5.22005E+07	1.00652	7450.76	7481.7	0.0041
Mn^{23+}	25	5.43887E+07	1.00733	8107.25	8140.6	0.0041
Fe^{24+}	26	5.65768E+07	1.00821	8792.66	8828	0.0040
Co^{25+}	27	5.87649E+07	1.00917	9507.25	9544.1	0.0039
Ni^{26+}	28	6.09529E+07	1.01022	10251.33	10288.8	0.0036
Cu^{27+}	29	6.31409E+07	1.01136	11025.21	11062.38	0.0034

[a] From Eq. (7.19) (Eq. (53)).

[b] From Eq. (7.29) (Eq. (61)).

[c] From Eq. (7.30).

[d] From Eq. (7.31).

[e] From Eq. (1.250) with the velocity given by Eq. (7.31).

[f] From Eqs. (7.28) and (7.47) with $E(electric)$ of Eq. (7.29) relativistically corrected by γ^* according to Eq.(1.251) except that the electron-nuclear electrodynamic relativistic factor corresponding to the reduced mass of Eqs. (1.213-1.223) was not included.

[g] From theoretical calculations for ions Ne^{8+} to Cu^{28+} [24-25].

[h] (Experimental-theoretical)/experimental.

APPROACH FOR THREE-THROUGH TWENTY-ELECTRON ATOMS

For each two-electron atom having a central charge of Z times that of the proton, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by Eq. (53). For $Z \geq 3$, the next electron which binds to form the corresponding three-electron atom is attracted by the central Coulomb field and is repelled by diamagnetic forces due to the spin-paired inner electrons such that it forms and unpaired orbitsphere at radius r_3. Since the charge-density function of each s electron including those of three-electron atoms is spherically symmetrical, the central Coulomb force, $\mathbf{F}_{ele}$, that acts on the outer electron to cause it to bind due to the nucleus and the inner electrons is given by

$$\mathbf{F}_{ele} = \frac{(Z-n)e^2}{4\pi\varepsilon_o r_n^2}\mathbf{i}_r \tag{58}$$

for $r > r_{n-1}$ where n corresponds to the number of electrons of the atom and Z is its atomic number. In each case, the magnetic field of the binding outer electron changes the angular velocities of the inner electrons. However, in each case, the magnetic field of the outer electron provides a central Lorentzian force which exactly balances the change in centrifugal force because of the change in angular velocity [4]. The inner electrons remain at their initial radii, but cause a diamagnetic force according to Lenz's law or a paramagnetic force depending on the spin and orbital angular momenta of the inner electrons and that of the outer. The force balance minimizes the energy of the atom.

It was shown previously [4] that the same principles including the central force given by Eq. (58) applies in the case that a nonuniform distribution of charge according to Eq. (14) achieves an energy minimum. In the case that an electron has orbital angular momentum in addition to spin angular momentum, the corresponding charge density wave is a time and spherical-harmonic wherein the traveling charge-density wave modulates the constant charge-density function as given in the Angular Functions section. It was found that electrostatic and magnetostatic s electrons pair in shells until a fifth electron is added. Then, a nonuniform distribution of charge achieves an energy minimum with the formation of a third shell due to the dependence of the magnetic forces on the nuclear charge and orbital energy (Eqs. (10.52), (10.55), and (10.93) of Ref. [4]). Minimum energy configurations are given by solutions to Laplace's equation. The general form of the solution is

$$\Phi(r,\theta,\phi)=\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell}B_{\ell,m}r^{-(\ell+1)}Y_{\ell}^{m}(\theta,\phi) \quad (59)$$

As demonstrated previously, this general solution also gives the functions of the resonant photons of excited states [4]. To maintain the symmetry of the central charge and the energy minimum condition given by solutions to Laplace's equation (Eq. (59)), the charge-density waves on electron orbitspheres at r_1 and r_3 complement those of the outer orbitals when the outer p orbitals are not all occupied by at least one electron, and the complementary charge-density waves are provided by electrons at r_3 when this condition is met. Since the angular harmonic charge-density waves are nonradiative as shown in the Nonradiation Based on the Electron Electromagnetic Fields and the Poynting Power Vector section, the time-averaged central field is inverse r-squared even though the central field is modulated by the concentric charge-density waves. The modulated central field maintains the spherical harmonic orbitals that maintain the spherical-harmonic phase according to Eq. (59). Thus, the central Coulomb force, $\mathbf{F}_{ele}$, that acts on the outer electron to cause it to bind due to the nucleus and the inner electrons is given by Eq. (58).

The outer electrons of atoms and ions that are isoelectronic with the series boron through neon half-fill a 2p level with unpaired electrons at nitrogen, then fill the level with paired electrons at neon. In general, electrons of an atom with the same principal and ℓ quantum numbers align parallel until each of the m_ℓ levels are occupied, and then pairing occurs until each of the m_ℓ levels contain paired electrons. The electron configuration for one through twenty-electron atoms that achieves an energy minimum is: 1s < 2s < 2p < 3s < 3p < 4s. In each case, the force balance of the central Coulombic, paramagnetic, and diamagnetic forces was derived for each n-electron atom that was solved for the radius of each electron. The ionization energies were obtained using the calculated radii in the determination of the Coulombic and any magnetic energies. The radii and ionization energies for all cases were given by equations having fundamental constants and each nuclear charge, Z, only. The predicted ionization energies and electron configurations compared with the experimental values [24-26] are given in TABLES I-XXIII.

The predicted electron configurations are in agreement with the experimental configurations known for 400 atoms and ions. The agreement between the experimental and calculated values of the ionization energies given in TABLES I-XX is well within the experimental capability of the spectroscopic determinations

including the values at large Z which relies on X-ray spectroscopy. Ionization energies are difficult to determine since the cut-off of the Rydberg series of lines at the ionization energy is often not observed. Thus, each series isoelectronic with the neutral n-electron atom given in TABLES I-XX [24-25] relies on theoretical calculations and interpolation of the isoelectronic and Rydberg series as well as direct experimental data to extend the precision beyond the capability of X-ray spectroscopy. But, no assurances can be given that these techniques are correct, and they may not improve the results. In each case, the error given in the last column of TABLES I-XX is very reasonable given the quality of the data.

TABLE III. Ionization energies for some three-electron atoms.

3 e Atom	Z	r_1 (a_o) a	r_3 (a_o) b	Electric Energy c (eV)	$\Delta\nu$ d (m/s)	ΔE_T e (eV)	Theoretical Ionization Energies f (eV)	Experimental Ionization Energies g (eV)	Relative Error h
Li	3	0.35566	2.55606	5.3230	1.6571E+04	1.5613E-03	5.40381	5.39172	-0.00224
Be^+	4	0.26116	1.49849	18.1594	4.4346E+04	1.1181E-02	18.1706	18.21116	0.00223
B^{2+}	5	0.20670	1.07873	37.8383	7.4460E+04	3.1523E-02	37.8701	37.93064	0.00160
C^{3+}	6	0.17113	0.84603	64.3278	1.0580E+05	6.3646E-02	64.3921	64.4939	0.00158
N^{4+}	7	0.14605	0.69697	97.6067	1.3782E+05	1.0800E-01	97.7160	97.8902	0.00178
O^{5+}	8	0.12739	0.59299	137.6655	1.7026E+05	1.6483E-01	137.8330	138.1197	0.00208
F^{6+}	9	0.11297	0.51621	184.5001	2.0298E+05	2.3425E-01	184.7390	185.186	0.00241
Ne^{7+}	10	0.10149	0.45713	238.1085	2.3589E+05	3.1636E-01	238.4325	239.0989	0.00279
Na^{8+}	11	0.09213	0.41024	298.4906	2.6894E+05	4.1123E-01	298.9137	299.864	0.00317
Mg^{9+}	12	0.08435	0.37210	365.6469	3.0210E+05	5.1890E-01	366.1836	367.5	0.00358
Al^{10+}	13	0.07778	0.34047	439.5790	3.3535E+05	6.3942E-01	440.2439	442	0.00397
Si^{11+}	14	0.07216	0.31381	520.2888	3.6868E+05	7.7284E-01	521.0973	523.42	0.00444
P^{12+}	15	0.06730	0.29102	607.7792	4.0208E+05	9.1919E-01	608.7469	611.74	0.00489
S^{13+}	16	0.06306	0.27132	702.0535	4.3554E+05	1.0785E+00	703.1966	707.01	0.00539
Cl^{14+}	17	0.05932	0.25412	803.1158	4.6905E+05	1.2509E+00	804.4511	809.4	0.00611
Ar^{15+}	18	0.05599	0.23897	910.9708	5.0262E+05	1.4364E+00	912.5157	918.03	0.00601
K^{16+}	19	0.05302	0.22552	1025.6241	5.3625E+05	1.6350E+00	1027.3967	1033.4	0.00581
Ca^{17+}	20	0.05035	0.21350	1147.0819	5.6993E+05	1.8468E+00	1149.1010	1157.8	0.00751
Sc^{18+}	21	0.04794	0.20270	1275.3516	6.0367E+05	2.0720E+00	1277.6367	1287.97	0.00802
Ti^{19+}	22	0.04574	0.19293	1410.4414	6.3748E+05	2.3106E+00	1413.0129	1425.4	0.00869
V^{20+}	23	0.04374	0.18406	1552.3606	6.7135E+05	2.5626E+00	1555.2398	1569.6	0.00915
Cr^{21+}	24	0.04191	0.17596	1701.1197	7.0530E+05	2.8283E+00	1704.3288	1721.4	0.00992
Mn^{22+}	25	0.04022	0.16854	1856.7301	7.3932E+05	3.1077E+00	1860.2926	1879.9	0.01043
Fe^{23+}	26	0.03867	0.16172	2019.205	7.734	3.4011E	2023.145	2023	-0.00007

				0	2E+05	+00	1		
Co^{24+}	27	0.03723	0.15542	2188.558	8.076	3.7084E	2192.902	2219	0.01176
				5	2E+05	+00	0		
Ni^{25+}	28	0.03589	0.14959	2364.806	8.419	4.0300E	2369.580	2399.2	0.01235
				5	1E+05	+00	3		
Cu^{26+}	29	0.03465	0.14418	2547.966	8.763	4.3661E	2553.198	2587.5	0.01326
				4	0E+05	+00	7		

[a] Radius of the paired inner electrons of three-electron atoms from Eq. (10.49) (Eq. (60)).

[b] Radius of the unpaired outer electron of three-electron atoms from Eq. (10.50) (Eq. (60)).

[c] Electric energy of the outer electron of three-electron atoms from Eq. (10.43) (Eq. (61)).

[d] Change in the velocity of the paired inner electrons due to the unpaired outer electron of three-electron atoms from Eq. (10.46).

[e] Change in the kinetic energy of the paired inner electrons due to the unpaired outer electron of three-electron atoms from Eq. (10.47).

[f] Calculated ionization energies of three-electron atoms from Eq. (10.48) for $Z > 3$ and Eq. (10.25) for Li.

[g] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[h] (Experimental-theoretical)/experimental.

TABLE IV. Ionization energies for some four-electron atoms.

4 e Atom	Z	r_1 (a_o) a	r_3 (a_o) b	Electric Energy c (eV)	Magnetic Energy d (eV)	$\Delta\nu$ e (m/s X 10^{-5})	ΔE_T f (eV)	Theoretical Ionization Energies g (eV)	Experimental Ionization Energies h (eV)	Relative Error i
Be	4	0.26116	1.52503	8.9178	0.03226	0.4207	0.0101	9.28430	9.32263	0.0041
B^{+}	5	0.20670	1.07930	25.2016	0.0910	0.7434	0.0314	25.1627	25.15484	-0.0003
C^{2+}	6	0.17113	0.84317	48.3886	0.1909	1.0688	0.0650	48.3125	47.8878	-0.0089
N^{3+}	7	0.14605	0.69385	78.4029	0.3425	1.3969	0.1109	78.2765	77.4735	-0.0104
O^{4+}	8	0.12739	0.59020	115.2148	0.5565	1.7269	0.1696	115.0249	113.899	-0.0099
F^{5+}	9	0.11297	0.51382	158.8102	0.8434	2.0582	0.2409	158.5434	157.1651	-0.0088
Ne^{6+}	10	0.10149	0.45511	209.1813	1.2138	2.3904	0.3249	208.8243	207.2759	-0.0075
Na^{7+}	11	0.09213	0.40853	266.3233	1.6781	2.7233	0.4217	265.8628	264.25	-0.0061
Mg^{8+}	12	0.08435	0.37065	330.2335	2.2469	3.0567	0.5312	329.6559	328.06	-0.0049
Al^{9+}	13	0.07778	0.33923	400.9097	2.9309	3.3905	0.6536	400.2017	398.75	-0.0036
Si^{10+}	14	0.07216	0.31274	478.3507	3.7404	3.7246	0.7888	477.4989	476.36	-0.0024
P^{11+}	15	0.06730	0.29010	562.5555	4.6861	4.0589	0.9367	561.5464	560.8	-0.0013
S^{12+}	16	0.06306	0.27053	653.5233	5.7784	4.3935	1.0975	652.3436	652.2	-0.0002
Cl^{13+}	17	0.05932	0.25344	751.2537	7.0280	4.7281	1.2710	749.8899	749.76	-0.0002
Ar^{14+}	18	0.05599	0.23839	855.7463	8.4454	5.0630	1.4574	854.1849	854.77	0.0007
K^{15+}	19	0.05302	0.22503	967.0007	10.0410	5.3979	1.6566	965.2283	968	0.0029
Ca^{16+}	20	0.05035	0.21308	1085.0167	11.8255	5.7329	1.8687	1083.0198	1087	0.0037
Sc^{17+}	21	0.04794	0.20235	1209.7940	13.8094	6.0680	2.0935	1207.5592	1213	0.0045
Ti^{18+}	22	0.04574	0.19264	1341.3326	16.0032	6.4032	2.3312	1338.8465	1346	0.0053
V^{19+}	23	0.04374	0.18383	1479.6323	18.4174	6.7384	2.5817	1476.8813	1486	0.0061
Cr^{20+}	24	0.04191	0.17579	1624.6929	21.0627	7.0737	2.8450	1621.6637	1634	0.0075
Mn^{21+}	25	0.04022	0.16842	1776.5144	23.9495	7.4091	3.1211	1773.1935	1788	0.0083
Fe^{22+}	26	0.03867	0.16165	1935.0968	27.0883	7.7444	3.4101	1931.4707	1950	0.0095
Co^{23+}	27	0.03723	0.15540	2100.4398	30.4898	8.0798	3.7118	2096.4952	2119	0.0106

Ni^{24+}	28	0.03589	0.14961	2272.5436	34.1644	8.4153	4.0264	2268.2669	2295	0.0116
Cu^{25+}	29	0.03465	0.14424	2451.4080	38.1228	8.7508	4.3539	2446.7858	2478	0.0126

[a] Radius of the paired inner electrons of four-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the paired outer electrons of four-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Electric energy of the outer electrons of four-electron atoms from Eq. (10.63) (Eq. (61)).

[d] Magnetic energy of the outer electrons of four-electron atoms upon unpairing from Eq. (7.30) and Eq. (10.64).

[e] Change in the velocity of the paired inner electrons due to the unpaired outer electron of four-electron atoms during ionization from Eq. (10.46).

[f] Change in the kinetic energy of the paired inner electrons due to the unpaired outer electron of four-electron atoms during ionization from Eq. (10.47).

[g] Calculated ionization energies of four-electron atoms from Eq. (10.68) for $Z > 4$ and Eq. (10.66) for Be.

[h] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[i] (Experimental-theoretical)/experimental.

TABLE V. Ionization energies for some five-electron atoms.

5 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_5 (a_o) [c]	Theoretical Ionization Energies [d] (eV)	Experimental Ionization Energies [e] (eV)	Relative Error [f]
B	5	0.20670	1.07930	1.67000	8.30266	8.29803	-0.00056
C^+	6	0.17113	0.84317	1.12092	24.2762	24.38332	0.0044
N^{2+}	7	0.14605	0.69385	0.87858	46.4585	47.44924	0.0209
O^{3+}	8	0.12739	0.59020	0.71784	75.8154	77.41353	0.0206
F^{4+}	9	0.11297	0.51382	0.60636	112.1922	114.2428	0.0179
Ne^{5+}	10	0.10149	0.45511	0.52486	155.5373	157.93	0.0152
Na^{6+}	11	0.09213	0.40853	0.46272	205.8266	208.5	0.0128
Mg^{7+}	12	0.08435	0.37065	0.41379	263.0469	265.96	0.0110
Al^{8+}	13	0.07778	0.33923	0.37425	327.1901	330.13	0.0089
Si^{9+}	14	0.07216	0.31274	0.34164	398.2509	401.37	0.0078
P^{10+}	15	0.06730	0.29010	0.31427	476.2258	479.46	0.0067
S^{11+}	16	0.06306	0.27053	0.29097	561.1123	564.44	0.0059
Cl^{12+}	17	0.05932	0.25344	0.27090	652.9086	656.71	0.0058
Ar^{13+}	18	0.05599	0.23839	0.25343	751.6132	755.74	0.0055
K^{14+}	19	0.05302	0.22503	0.23808	857.2251	861.1	0.0045
Ca^{15+}	20	0.05035	0.21308	0.22448	969.7435	974	0.0044
Sc^{16+}	21	0.04794	0.20235	0.21236	1089.1678	1094	0.0044
Ti^{17+}	22	0.04574	0.19264	0.20148	1215.4975	1221	0.0045
V^{18+}	23	0.04374	0.18383	0.19167	1348.7321	1355	0.0046
Cr^{19+}	24	0.04191	0.17579	0.18277	1488.8713	1496	0.0048
Mn^{20+}	25	0.04022	0.16842	0.17466	1635.9148	1644	0.0049
Fe^{21+}	26	0.03867	0.16165	0.16724	1789.8624	1799	0.0051
Co^{22+}	27	0.03723	0.15540	0.16042	1950.7139	1962	0.0058
Ni^{23+}	28	0.03589	0.14961	0.15414	2118.4690	2131	0.0059
Cu^{24+}	29	0.03465	0.14424	0.14833	2293.1278	2308	0.0064

[a] Radius of the first set of paired inner electrons of five-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the second set of paired inner electrons of five-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the outer electron of five-electron atoms from Eq. (10.113) (Eq. (64)) for $Z > 5$ and Eq. (10.101) for B.

[d] Calculated ionization energies of five-electron atoms given by the electric energy (Eq. (10.114)) (Eq. (61)) for $Z > 5$ and Eq. (10.104) for B.

[e] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[f] (Experimental-theoretical)/experimental.

TABLE VI. Ionization energies for some six-electron atoms.

6 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_6 (a_o) [c]	Theoretical Ionization Energies [d] (eV)	Experimental Ionization Energies [e] (eV)	Relative Error [f]
C	6	0.17113	0.84317	1.20654	11.27671	11.2603	-0.0015
N^{+}	7	0.14605	0.69385	0.90119	30.1950	29.6013	-0.0201
O^{2+}	8	0.12739	0.59020	0.74776	54.5863	54.9355	0.0064
F^{3+}	9	0.11297	0.51382	0.63032	86.3423	87.1398	0.0092
Ne^{4+}	10	0.10149	0.45511	0.54337	125.1986	126.21	0.0080
Na^{5+}	11	0.09213	0.40853	0.47720	171.0695	172.18	0.0064
Mg^{6+}	12	0.08435	0.37065	0.42534	223.9147	225.02	0.0049
Al^{7+}	13	0.07778	0.33923	0.38365	283.7121	284.66	0.0033
Si^{8+}	14	0.07216	0.31274	0.34942	350.4480	351.12	0.0019
P^{9+}	15	0.06730	0.29010	0.32081	424.1135	424.4	0.0007
S^{10+}	16	0.06306	0.27053	0.29654	504.7024	504.8	0.0002
Cl^{11+}	17	0.05932	0.25344	0.27570	592.2103	591.99	-0.0004
Ar^{12+}	18	0.05599	0.23839	0.25760	686.6340	686.1	-0.0008
K^{13+}	19	0.05302	0.22503	0.24174	787.9710	786.6	-0.0017
Ca^{14+}	20	0.05035	0.21308	0.22772	896.2196	894.5	-0.0019
Sc^{15+}	21	0.04794	0.20235	0.21524	1011.3782	1009	-0.0024
Ti^{16+}	22	0.04574	0.19264	0.20407	1133.4456	1131	-0.0022
V^{17+}	23	0.04374	0.18383	0.19400	1262.4210	1260	-0.0019
Cr^{18+}	24	0.04191	0.17579	0.18487	1398.3036	1396	-0.0017
Mn^{19+}	25	0.04022	0.16842	0.17657	1541.0927	1539	-0.0014
Fe^{20+}	26	0.03867	0.16165	0.16899	1690.7878	1689	-0.0011
Co^{21+}	27	0.03723	0.15540	0.16203	1847.3885	1846	-0.0008
Ni^{22+}	28	0.03589	0.14961	0.15562	2010.8944	2011	0.0001
Cu^{23+}	29	0.03465	0.14424	0.14970	2181.3053	2182	0.0003

[a] Radius of the first set of paired inner electrons of six-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the second set of paired inner electrons of six-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the two unpaired outer electrons of six-electron atoms from Eq. (10.132) (Eq. (64)) for $Z > 6$ and Eq. (10.122) for C.

[d] Calculated ionization energies of six-electron atoms given by the electric energy (Eq. (10.133)) (Eq. (61)).

[e] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[f] (Experimental-theoretical)/experimental.

TABLE VII. Ionization energies for some seven-electron atoms.

7 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_7 (a_o) [c]	Theoretical Ionization Energies [d] (eV)	Experimental Ionization Energies [e] (eV)	Relative Error [f]
N	7	0.14605	0.69385	0.93084	14.61664	14.53414	-0.0057
O^+	8	0.12739	0.59020	0.78489	34.6694	35.1173	0.0128
F^{2+}	9	0.11297	0.51382	0.67084	60.8448	62.7084	0.0297
Ne^{3+}	10	0.10149	0.45511	0.57574	94.5279	97.12	0.0267
Na^{4+}	11	0.09213	0.40853	0.50250	135.3798	138.4	0.0218
Mg^{5+}	12	0.08435	0.37065	0.44539	183.2888	186.76	0.0186
Al^{6+}	13	0.07778	0.33923	0.39983	238.2017	241.76	0.0147
Si^{7+}	14	0.07216	0.31274	0.36271	300.0883	303.54	0.0114
P^{8+}	15	0.06730	0.29010	0.33191	368.9298	372.13	0.0086
S^{9+}	16	0.06306	0.27053	0.30595	444.7137	447.5	0.0062
Cl^{10+}	17	0.05932	0.25344	0.28376	527.4312	529.28	0.0035
Ar^{11+}	18	0.05599	0.23839	0.26459	617.0761	618.26	0.0019
K^{12+}	19	0.05302	0.22503	0.24785	713.6436	714.6	0.0013
Ca^{13+}	20	0.05035	0.21308	0.23311	817.1303	817.6	0.0006
Sc^{14+}	21	0.04794	0.20235	0.22003	927.5333	927.5	0.0000
Ti^{15+}	22	0.04574	0.19264	0.20835	1044.8504	1044	-0.0008
V^{16+}	23	0.04374	0.18383	0.19785	1169.0800	1168	-0.0009
Cr^{17+}	24	0.04191	0.17579	0.18836	1300.2206	1299	-0.0009
Mn^{18+}	25	0.04022	0.16842	0.17974	1438.2710	1437	-0.0009
Fe^{19+}	26	0.03867	0.16165	0.17187	1583.2303	1582	-0.0008
Co^{20+}	27	0.03723	0.15540	0.16467	1735.0978	1735	-0.0001
Ni^{21+}	28	0.03589	0.14961	0.15805	1893.8726	1894	0.0001
Cu^{22+}	29	0.03465	0.14424	0.15194	2059.5543	2060	0.0002

[a] Radius of the first set of paired inner electrons of seven-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the second set of paired inner electrons of seven-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the three unpaired paired outer electrons of seven-electron atoms from Eq. (10.152) (Eq. (64)) for $Z > 7$ and Eq. (10.142) for N.

[d] Calculated ionization energies of seven-electron atoms given by the electric energy (Eq. (10.153)) (Eq. (61)).

[e] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[f] (Experimental-theoretical)/experimental.

TABLE VIII. Ionization energies for some eight-electron atoms.

8 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_8 (a_o) [c]	Theoretical Ionization Energies [d] (eV)	Experimental Ionization Energies [e] (eV)	Relative Error [f]
O	8	0.12739	0.59020	1.00000	13.60580	13.6181	0.0009
F^+	9	0.11297	0.51382	0.7649	35.5773	34.9708	-0.0173
Ne^{2+}	10	0.10149	0.45511	0.6514	62.6611	63.45	0.0124
Na^{3+}	11	0.09213	0.40853	0.5592	97.3147	98.91	0.0161
Mg^{4+}	12	0.08435	0.37065	0.4887	139.1911	141.27	0.0147
Al^{5+}	13	0.07778	0.33923	0.4338	188.1652	190.49	0.0122
Si^{6+}	14	0.07216	0.31274	0.3901	244.1735	246.5	0.0094
P^{7+}	15	0.06730	0.29010	0.3543	307.1791	309.6	0.0078
S^{8+}	16	0.06306	0.27053	0.3247	377.1579	379.55	0.0063
Cl^{9+}	17	0.05932	0.25344	0.2996	454.0940	455.63	0.0034
Ar^{10+}	18	0.05599	0.23839	0.2782	537.9756	538.96	0.0018
K^{11+}	19	0.05302	0.22503	0.2597	628.7944	629.4	0.0010
Ca^{12+}	20	0.05035	0.21308	0.2434	726.5442	726.6	0.0001
Sc^{13+}	21	0.04794	0.20235	0.2292	831.2199	830.8	-0.0005
Ti^{14+}	22	0.04574	0.19264	0.2165	942.8179	941.9	-0.0010
V^{15+}	23	0.04374	0.18383	0.2051	1061.3351	1060	-0.0013
Cr^{16+}	24	0.04191	0.17579	0.1949	1186.7691	1185	-0.0015
Mn^{17+}	25	0.04022	0.16842	0.1857	1319.1179	1317	-0.0016
Fe^{18+}	26	0.03867	0.16165	0.1773	1458.3799	1456	-0.0016
Co^{19+}	27	0.03723	0.15540	0.1696	1604.5538	1603	-0.0010
Ni^{20+}	28	0.03589	0.14961	0.1626	1757.6383	1756	-0.0009
Cu^{21+}	29	0.03465	0.14424	0.1561	1917.6326	1916	-0.0009

[a] Radius of the first set of paired inner electrons of eight-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the second set of paired inner electrons of eight-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the two paired and two unpaired outer electrons of eight-electron atoms from Eq. (10.172) (Eq. (64)) for $Z > 8$ and Eq. (10.162) for O.

[d] Calculated ionization energies of eight-electron atoms given by the electric energy (Eq. (10.173)) (Eq. (61)).

[e] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[f] (Experimental-theoretical)/experimental.

TABLE IX. Ionization energies for some nine-electron atoms.

9 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_9 (a_o) [c]	Theoretical Ionization Energies [d] (eV)	Experimental Ionization Energies [e] (eV)	Relative Error [f]
F	9	0.11297	0.51382	0.78069	17.42782	17.42282	-0.0003
Ne^{+}	10	0.10149	0.45511	0.64771	42.0121	40.96328	-0.0256
Na^{2+}	11	0.09213	0.40853	0.57282	71.2573	71.62	0.0051
Mg^{3+}	12	0.08435	0.37065	0.50274	108.2522	109.2655	0.0093
Al^{4+}	13	0.07778	0.33923	0.44595	152.5469	153.825	0.0083
Si^{5+}	14	0.07216	0.31274	0.40020	203.9865	205.27	0.0063
P^{6+}	15	0.06730	0.29010	0.36283	262.4940	263.57	0.0041
S^{7+}	16	0.06306	0.27053	0.33182	328.0238	328.75	0.0022
Cl^{8+}	17	0.05932	0.25344	0.30571	400.5466	400.06	-0.0012
Ar^{9+}	18	0.05599	0.23839	0.28343	480.0424	478.69	-0.0028
K^{10+}	19	0.05302	0.22503	0.26419	566.4968	564.7	-0.0032
Ca^{11+}	20	0.05035	0.21308	0.24742	659.8992	657.2	-0.0041
Sc^{12+}	21	0.04794	0.20235	0.23266	760.2415	756.7	-0.0047
Ti^{13+}	22	0.04574	0.19264	0.21957	867.5176	863.1	-0.0051
V^{14+}	23	0.04374	0.18383	0.20789	981.7224	976	-0.0059
Cr^{15+}	24	0.04191	0.17579	0.19739	1102.8523	1097	-0.0053
Mn^{16+}	25	0.04022	0.16842	0.18791	1230.9038	1224	-0.0056
Fe^{17+}	26	0.03867	0.16165	0.17930	1365.8746	1358	-0.0058
Co^{18+}	27	0.03723	0.15540	0.17145	1507.7624	1504.6	-0.0021
Ni^{19+}	28	0.03589	0.14961	0.16427	1656.5654	1648	-0.0052
Cu^{20+}	29	0.03465	0.14424	0.15766	1812.2821	1804	-0.0046

[a] Radius of the first set of paired inner electrons of nine-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the second set of paired inner electrons of nine-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the one unpaired and two sets of paired outer electrons of nine-electron atoms from Eq. (10.192) (Eq. (64)) for $Z > 9$ and Eq. (10.182) for F.

[d] Calculated ionization energies of nine-electron atoms given by the electric energy (Eq. (10.193)) (Eq. (61)).

[e] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[f] (Experimental-theoretical)/experimental.

TABLE X. Ionization energies for some ten-electron atoms.

10 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	Theoretical Ionization Energies [d] (eV)	Experimental Ionization Energies [e] (eV)	Relative Error [f]
Ne	10	0.10149	0.45511	0.63659	21.37296	21.56454	0.00888
Na^+	11	0.09213	0.40853	0.560945	48.5103	47.2864	-0.0259
Mg^{2+}	12	0.08435	0.37065	0.510568	79.9451	80.1437	0.0025
Al^{3+}	13	0.07778	0.33923	0.456203	119.2960	119.992	0.0058
Si^{4+}	14	0.07216	0.31274	0.409776	166.0150	166.767	0.0045
P^{5+}	15	0.06730	0.29010	0.371201	219.9211	220.421	0.0023
S^{6+}	16	0.06306	0.27053	0.339025	280.9252	280.948	0.0001
Cl^{7+}	17	0.05932	0.25344	0.311903	348.9750	348.28	-0.0020
Ar^{8+}	18	0.05599	0.23839	0.288778	424.0365	422.45	-0.0038
K^{9+}	19	0.05302	0.22503	0.268844	506.0861	503.8	-0.0045
Ca^{10+}	20	0.05035	0.21308	0.251491	595.1070	591.9	-0.0054
Sc^{11+}	21	0.04794	0.20235	0.236251	691.0866	687.36	-0.0054
Ti^{12+}	22	0.04574	0.19264	0.222761	794.0151	787.84	-0.0078
V^{13+}	23	0.04374	0.18383	0.210736	903.8853	896	-0.0088
Cr^{14+}	24	0.04191	0.17579	0.19995	1020.6910	1010.6	-0.0100
Mn^{15+}	25	0.04022	0.16842	0.19022	1144.4276	1134.7	-0.0086
Fe^{16+}	26	0.03867	0.16165	0.181398	1275.0911	1266	-0.0072
Co^{17+}	27	0.03723	0.15540	0.173362	1412.6783	1397.2	-0.0111
Ni^{18+}	28	0.03589	0.14961	0.166011	1557.1867	1541	-0.0105
Cu^{19+}	29	0.03465	0.14424	0.159261	1708.6139	1697	-0.0068
Zn^{20+}	30	0.03349	0.13925	0.153041	1866.9581	1856	-0.0059

[a] Radius of the first set of paired inner electrons of ten-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the second set of paired inner electrons of ten-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of three sets of paired outer electrons of ten-electron atoms from Eq. (10.212)) (Eq. (64)) for $Z > 10$ and Eq. (10.202) for Ne.

[d] Calculated ionization energies of ten-electron atoms given by the electric energy (Eq. (10.213)) (Eq. (61)).

[e] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[f] (Experimental-theoretical)/experimental.

TABLE XI. Ionization energies for some eleven-electron atoms.

11 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	r_{11} (a_o) [d]	Theoretical Ionization Energies [e] (eV)	Experimental Ionization Energies [f] (eV)	Relative Error [g]
Na	11	0.09213	0.40853	0.560945	2.65432	5.12592	5.13908	0.0026
Mg^+	12	0.08435	0.37065	0.510568	1.74604	15.5848	15.03528	-0.0365
Al^{2+}	13	0.07778	0.33923	0.456203	1.47399	27.6918	28.44765	0.0266
Si^{3+}	14	0.07216	0.31274	0.409776	1.25508	43.3624	45.14181	0.0394
P^{4+}	15	0.06730	0.29010	0.371201	1.08969	62.4299	65.0251	0.0399
S^{5+}	16	0.06306	0.27053	0.339025	0.96226	84.8362	88.0530	0.0365
Cl^{6+}	17	0.05932	0.25344	0.311903	0.86151	110.5514	114.1958	0.0319
Ar^{7+}	18	0.05599	0.23839	0.288778	0.77994	139.5577	143.460	0.0272
K^{8+}	19	0.05302	0.22503	0.268844	0.71258	171.8433	175.8174	0.0226
Ca^{9+}	20	0.05035	0.21308	0.251491	0.65602	207.3998	211.275	0.0183
Sc^{10+}	21	0.04794	0.20235	0.236251	0.60784	246.2213	249.798	0.0143
Ti^{11+}	22	0.04574	0.19264	0.222761	0.56631	288.3032	291.500	0.0110
V^{12+}	23	0.04374	0.18383	0.210736	0.53014	333.6420	336.277	0.0078
Cr^{13+}	24	0.04191	0.17579	0.19995	0.49834	382.2350	384.168	0.0050
Mn^{14+}	25	0.04022	0.16842	0.19022	0.47016	434.0801	435.163	0.0025
Fe^{15+}	26	0.03867	0.16165	0.181398	0.44502	489.1753	489.256	0.0002
Co^{16+}	27	0.03723	0.15540	0.173362	0.42245	547.5194	546.58	-0.0017
Ni^{17+}	28	0.03589	0.14961	0.166011	0.40207	609.1111	607.06	-0.0034
Cu^{18+}	29	0.03465	0.14424	0.159261	0.38358	673.9495	670.588	-0.0050
Zn^{19+}	30	0.03349	0.13925	0.153041	0.36672	742.0336	738	-0.0055

[a] Radius of the first set of paired inner electrons of eleven-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the second set of paired inner electrons of eleven-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of three sets of paired inner electrons of eleven-electron atoms from Eq. (10.212)) (Eq. (64)).

[d] Radius of unpaired outer electron of eleven-electron atoms from Eq. (10.235)) (Eq. (60)) for $Z > 11$ and Eq. (10.226) for Na.

[e] Calculated ionization energies of eleven-electron atoms given by the electric energy (Eq. (10.236)) (Eq. (61)).

[f] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[g] (Experimental-theoretical)/experimental.

TABLE XII. Ionization energies for some twelve-electron atoms.

12 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	r_{12} (a_o) [d]	Theoretical Ionization Energies [e] (eV)	Experimental Ionization Energies [f] (eV)	Relative Error [g]
Mg	12	0.08435	0.37065	0.51057	1.79386	7.58467	7.64624	0.0081
Al^{+}	13	0.07778	0.33923	0.45620	1.41133	19.2808	18.82856	-0.0240
Si^{2+}	14	0.07216	0.31274	0.40978	1.25155	32.6134	33.49302	0.0263
P^{3+}	15	0.06730	0.29010	0.37120	1.09443	49.7274	51.4439	0.0334
S^{4+}	16	0.06306	0.27053	0.33902	0.96729	70.3296	72.5945	0.0312
Cl^{5+}	17	0.05932	0.25344	0.31190	0.86545	94.3266	97.03	0.0279
Ar^{6+}	18	0.05599	0.23839	0.28878	0.78276	121.6724	124.323	0.0213
K^{7+}	19	0.05302	0.22503	0.26884	0.71450	152.3396	154.88	0.0164
Ca^{8+}	20	0.05035	0.21308	0.25149	0.65725	186.3102	188.54	0.0118
Sc^{9+}	21	0.04794	0.20235	0.23625	0.60857	223.5713	225.18	0.0071
Ti^{10+}	22	0.04574	0.19264	0.22276	0.56666	264.1138	265.07	0.0036
V^{11+}	23	0.04374	0.18383	0.21074	0.53022	307.9304	308.1	0.0006
Cr^{12+}	24	0.04191	0.17579	0.19995	0.49822	355.0157	354.8	-0.0006
Mn^{13+}	25	0.04022	0.16842	0.19022	0.46990	405.3653	403.0	-0.0059
Fe^{14+}	26	0.03867	0.16165	0.18140	0.44466	458.9758	457	-0.0043
Co^{15+}	27	0.03723	0.15540	0.17336	0.42201	515.8442	511.96	-0.0076
Ni^{16+}	28	0.03589	0.14961	0.16601	0.40158	575.9683	571.08	-0.0086
Cu^{17+}	29	0.03465	0.14424	0.15926	0.38305	639.3460	633	-0.0100
Zn^{18+}	30	0.03349	0.13925	0.15304	0.36617	705.9758	698	-0.0114

[a] Radius of the first set of paired inner electrons of twelve-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the second set of paired inner electrons of twelve-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of three sets of paired inner electrons of twelve-electron atoms from Eq. (10.212)) (Eq. (64)).

[d] Radius of paired outer electrons of twelve-electron atoms from Eq. (10.255)) (Eq. (60)) for $Z>12$ and Eq. (10.246) for Mg.

[e] Calculated ionization energies of twelve-electron atoms given by the electric energy (Eq. (10.256)) (Eq. (61)).

[f] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[g] (Experimental-theoretical)/experimental.

TABLE XIII. Ionization energies for some thirteen-electron atoms.

13 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	r_{12} (a_o) [d]	r_{13} (a_o) [e]	Theoretical Ionization Energies [f] (eV)	Experimental Ionization Energies [g] (eV)	Relative Error [h]
Al	13	0.07778	0.33923	0.45620	1.41133	2.28565	5.98402	5.98577	0.0003
Si^{+}	14	0.07216	0.31274	0.40978	1.25155	1.5995	17.0127	16.34585	-0.0408
P^{2+}	15	0.06730	0.29010	0.37120	1.09443	1.3922	29.3195	30.2027	0.0292
S^{3+}	16	0.06306	0.27053	0.33902	0.96729	1.1991	45.3861	47.222	0.0389
Cl^{4+}	17	0.05932	0.25344	0.31190	0.86545	1.0473	64.9574	67.8	0.0419
Ar^{5+}	18	0.05599	0.23839	0.28878	0.78276	0.9282	87.9522	91.009	0.0336
K^{6+}	19	0.05302	0.22503	0.26884	0.71450	0.8330	114.3301	117.56	0.0275
Ca^{7+}	20	0.05035	0.21308	0.25149	0.65725	0.7555	144.0664	147.24	0.0216
Sc^{8+}	21	0.04794	0.20235	0.23625	0.60857	0.6913	177.1443	180.03	0.0160
Ti^{9+}	22	0.04574	0.19264	0.22276	0.56666	0.6371	213.5521	215.92	0.0110
V^{10+}	23	0.04374	0.18383	0.21074	0.53022	0.5909	253.2806	255.7	0.0095
Cr^{11+}	24	0.04191	0.17579	0.19995	0.49822	0.5510	296.3231	298.0	0.0056
Mn^{12+}	25	0.04022	0.16842	0.19022	0.46990	0.5162	342.6741	343.6	0.0027
Fe^{13+}	26	0.03867	0.16165	0.18140	0.44466	0.4855	392.3293	392.2	-0.0003
Co^{14+}	27	0.03723	0.15540	0.17336	0.42201	0.4583	445.2849	444	-0.0029
Ni^{15+}	28	0.03589	0.14961	0.16601	0.40158	0.4341	501.5382	499	-0.0051
Cu^{16+}	29	0.03465	0.14424	0.15926	0.38305	0.4122	561.0867	557	-0.0073
Zn^{17+}	30	0.03349	0.13925	0.15304	0.36617	0.3925	623.9282	619	-0.0080

[a] Radius of the paired 1s inner electrons of thirteen-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the paired 2s inner electrons of thirteen-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the three sets of paired 2p inner electrons of thirteen-electron atoms from Eq. (10.212)) (Eq. (64)).

[d] Radius of the paired 3s inner electrons of thirteen-electron atoms from Eq. (10.255)) (Eq. (60)).

[e] Radius of the unpaired 3p outer electron of thirteen-electron atoms from Eq. (10.288) (Eq. (67)) for $Z > 13$ and Eq. (10.276) for Al.

[f] Calculated ionization energies of thirteen-electron atoms given by the electric energy (Eq. (10.289)) (Eq. (61)) for $Z > 13$ and Eq. (10.279) for Al.

[g] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[h] (Experimental-theoretical)/experimental.

TABLE XIV. Ionization energies for some fourteen-electron atoms.

14 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	r_{12} (a_o) [d]	r_{14} (a_o) [e]	Theoretical Ionization Energies [f] (eV)	Experimental Ionization Energies [g] (eV)	Relative Error [h]
Si	14	0.07216	0.31274	0.40978	1.25155	1.67685	8.11391	8.15169	0.0046
P^+	15	0.06730	0.29010	0.37120	1.09443	1.35682	20.0555	19.7694	-0.0145
S^{2+}	16	0.06306	0.27053	0.33902	0.96729	1.21534	33.5852	34.790	0.0346
Cl^{3+}	17	0.05932	0.25344	0.31190	0.86545	1.06623	51.0426	53.4652	0.0453
Ar^{4+}	18	0.05599	0.23839	0.28878	0.78276	0.94341	72.1094	75.020	0.0388
K^{5+}	19	0.05302	0.22503	0.26884	0.71450	0.84432	96.6876	99.4	0.0273
Ca^{6+}	20	0.05035	0.21308	0.25149	0.65725	0.76358	124.7293	127.2	0.0194
Sc^{7+}	21	0.04794	0.20235	0.23625	0.60857	0.69682	156.2056	158.1	0.0120
Ti^{8+}	22	0.04574	0.19264	0.22276	0.56666	0.64078	191.0973	192.10	0.0052
V^{9+}	23	0.04374	0.18383	0.21074	0.53022	0.59313	229.3905	230.5	0.0048
Cr^{10+}	24	0.04191	0.17579	0.19995	0.49822	0.55211	271.0748	270.8	-0.0010
Mn^{11+}	25	0.04022	0.16842	0.19022	0.46990	0.51644	316.1422	314.4	-0.0055
Fe^{12+}	26	0.03867	0.16165	0.18140	0.44466	0.48514	364.5863	361	-0.0099
Co^{13+}	27	0.03723	0.15540	0.17336	0.42201	0.45745	416.4021	411	-0.0131
Ni^{14+}	28	0.03589	0.14961	0.16601	0.40158	0.43277	471.5854	464	-0.0163
Cu^{15+}	29	0.03465	0.14424	0.15926	0.38305	0.41064	530.1326	520	-0.0195
Zn^{16+}	30	0.03349	0.13925	0.15304	0.36617	0.39068	592.0410	579	-0.0225

[a] Radius of the paired 1s inner electrons of fourteen-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the paired 2s inner electrons of fourteen-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the three sets of paired 2p inner electrons of fourteen-electron atoms from Eq. (10.212)) (Eq. (64)).

[d] Radius of the paired 3s inner electrons of fourteen-electron atoms from Eq. (10.255)) (Eq. (60)).

[e] Radius of the two unpaired 3p outer electrons of fourteen-electron atoms from Eq. (10.309) (Eq. (67)) for $Z > 14$ and Eq. (10.297) for Si.

[f] Calculated ionization energies of fourteen-electron atoms given by the electric energy (Eq. (10.310)) (Eq. (61)).

[g] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[h] (Experimental-theoretical)/experimental.

TABLE XV. Ionization energies for some fifteen-electron atoms.

15 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	r_{12} (a_o) [d]	r_{15} (a_o) [e]	Theoretical Ionization Energies [f] (eV)	Experimental Ionization Energies [g] (eV)	Relative Error [h]
P	15	0.06730	0.29010	0.37120	1.09443	1.28900	10.55536	10.48669	-0.0065
S^+	16	0.06306	0.27053	0.33902	0.96729	1.15744	23.5102	23.3379	-0.0074
Cl^{2+}	17	0.05932	0.25344	0.31190	0.86545	1.06759	38.2331	39.61	0.0348
Ar^{3+}	18	0.05599	0.23839	0.28878	0.78276	0.95423	57.0335	59.81	0.0464
K^{4+}	19	0.05302	0.22503	0.26884	0.71450	0.85555	79.5147	82.66	0.0381
Ca^{5+}	20	0.05035	0.21308	0.25149	0.65725	0.77337	105.5576	108.78	0.0296
Sc^{6+}	21	0.04794	0.20235	0.23625	0.60857	0.70494	135.1046	138.0	0.0210
Ti^{7+}	22	0.04574	0.19264	0.22276	0.56666	0.64743	168.1215	170.4	0.0134
V^{8+}	23	0.04374	0.18383	0.21074	0.53022	0.59854	204.5855	205.8	0.0059
Cr^{9+}	24	0.04191	0.17579	0.19995	0.49822	0.55652	244.4799	244.4	-0.0003
Mn^{10+}	25	0.04022	0.16842	0.19022	0.46990	0.52004	287.7926	286.0	-0.0063
Fe^{11+}	26	0.03867	0.16165	0.18140	0.44466	0.48808	334.5138	330.8	-0.0112
Co^{12+}	27	0.03723	0.15540	0.17336	0.42201	0.45985	384.6359	379	-0.0149
Ni^{13+}	28	0.03589	0.14961	0.16601	0.40158	0.43474	438.1529	430	-0.0190
Cu^{14+}	29	0.03465	0.14424	0.15926	0.38305	0.41225	495.0596	484	-0.0229
Zn^{15+}	30	0.03349	0.13925	0.15304	0.36617	0.39199	555.3519	542	-0.0246

[a] Radius of the paired 1s inner electrons of fifteen-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the paired 2s inner electrons of fifteen-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the three sets of paired 2p inner electrons of fifteen-electron atoms from Eq. (10.212)) (Eq. (64)).

[d] Radius of the paired 3s inner electrons of fifteen-electron atoms from Eq. (10.255)) (Eq. (60)).

[e] Radius of the three unpaired 3p outer electrons of fifteen-electron atoms from Eq. (10.331) (Eq. (67)) for $Z>15$ and Eq. (10.319) for P.

[f] Calculated ionization energies of fifteen-electron atoms given by the electric energy (Eq. (10.332)) (Eq. (61)).

[g] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[h] (Experimental-theoretical)/experimental.

TABLE XVI. Ionization energies for some sixteen-electron atoms.

16 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	r_{12} (a_o) [d]	r_{16} (a_o) [e]	Theoretical Ionization Energies [f] (eV)	Experimental Ionization Energies [g] (eV)	Relative Error [h]
S	16	0.06306	0.27053	0.33902	0.96729	1.32010	10.30666	10.36001	0.0051
Cl^+	17	0.05932	0.25344	0.31190	0.86545	1.10676	24.5868	23.814	-0.0324
Ar^{2+}	18	0.05599	0.23839	0.28878	0.78276	1.02543	39.8051	40.74	0.0229
K^{3+}	19	0.05302	0.22503	0.26884	0.71450	0.92041	59.1294	60.91	0.0292
Ca^{4+}	20	0.05035	0.21308	0.25149	0.65725	0.82819	82.1422	84.50	0.0279
Sc^{5+}	21	0.04794	0.20235	0.23625	0.60857	0.75090	108.7161	110.68	0.0177
Ti^{6+}	22	0.04574	0.19264	0.22276	0.56666	0.68622	138.7896	140.8	0.0143
V^{7+}	23	0.04374	0.18383	0.21074	0.53022	0.63163	172.3256	173.4	0.0062
Cr^{8+}	24	0.04191	0.17579	0.19995	0.49822	0.58506	209.2996	209.3	0.0000
Mn^{9+}	25	0.04022	0.16842	0.19022	0.46990	0.54490	249.6938	248.3	-0.0056
Fe^{10+}	26	0.03867	0.16165	0.18140	0.44466	0.50994	293.4952	290.2	-0.0114
Co^{11+}	27	0.03723	0.15540	0.17336	0.42201	0.47923	340.6933	336	-0.0140
Ni^{12+}	28	0.03589	0.14961	0.16601	0.40158	0.45204	391.2802	384	-0.0190
Cu^{13+}	29	0.03465	0.14424	0.15926	0.38305	0.42781	445.2492	435	-0.0236
Zn^{14+}	30	0.03349	0.13925	0.15304	0.36617	0.40607	502.5950	490	-0.0257

[a] Radius of the paired 1s inner electrons of sixteen-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the paired 2s inner electrons of sixteen-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the three sets of paired 2p inner electrons of sixteen-electron atoms from Eq. (10.212)) (Eq. (64)).

[d] Radius of the paired 3s inner electrons of sixteen-electron atoms from Eq. (10.255)) (Eq. (60)).

[e] Radius of the two paired and two unpaired 3p outer electrons of sixteen-electron atoms from Eq. (10.353) (Eq. (67)) for $Z > 16$ and Eq. (10.341) for S.

[f] Calculated ionization energies of sixteen-electron atoms given by the electric energy (Eq. (10.354)) (Eq. (61)).

[g] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[h] (Experimental-theoretical)/experimental.

TABLE XVII. Ionization energies for some seventeen-electron atoms.

17 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	r_{12} (a_o) [d]	r_{17} (a_o) [e]	Theoretical Ionization Energies [f] (eV)	Experimental Ionization Energies [g] (eV)	Relative Error [h]
Cl	17	0.05932	0.25344	0.31190	0.86545	1.05158	12.93841	12.96764	0.0023
Ar^+	18	0.05599	0.23839	0.28878	0.78276	0.98541	27.6146	27.62967	0.0005
K^{2+}	19	0.05302	0.22503	0.26884	0.71450	0.93190	43.8001	45.806	0.0438
Ca^{3+}	20	0.05035	0.21308	0.25149	0.65725	0.84781	64.1927	67.27	0.0457
Sc^{4+}	21	0.04794	0.20235	0.23625	0.60857	0.77036	88.3080	91.65	0.0365
Ti^{5+}	22	0.04574	0.19264	0.22276	0.56666	0.70374	116.0008	119.53	0.0295
V^{6+}	23	0.04374	0.18383	0.21074	0.53022	0.64701	147.2011	150.6	0.0226
Cr^{7+}	24	0.04191	0.17579	0.19995	0.49822	0.59849	181.8674	184.7	0.0153
Mn^{8+}	25	0.04022	0.16842	0.19022	0.46990	0.55667	219.9718	221.8	0.0082
Fe^{9+}	26	0.03867	0.16165	0.18140	0.44466	0.52031	261.4942	262.1	0.0023
Co^{10+}	27	0.03723	0.15540	0.17336	0.42201	0.48843	306.4195	305	-0.0047
Ni^{11+}	28	0.03589	0.14961	0.16601	0.40158	0.46026	354.7360	352	-0.0078
Cu^{12+}	29	0.03465	0.14424	0.15926	0.38305	0.43519	406.4345	401	-0.0136
Zn^{13+}	30	0.03349	0.13925	0.15304	0.36617	0.41274	461.5074	454	-0.0165

[a] Radius of the paired 1s inner electrons of seventeen-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the paired 2s inner electrons of seventeen-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the three sets of paired 2p inner electrons of seventeen-electron atoms from Eq. (10.212)) (Eq. (64)).

[d] Radius of the paired 3s inner electrons of seventeen-electron atoms from Eq. (10.255)) (Eq. (60)).

[e] Radius of the two sets of paired and an unpaired 3p outer electron of seventeen-electron atoms from Eq. (10.376) (Eq. (67)) for $Z > 17$ and Eq. (10.363) for Cl.

[f] Calculated ionization energies of seventeen-electron atoms given by the electric energy (Eq. (10.377)) (Eq. (61)).

[g] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[h] (Experimental-theoretical)/experimental.

TABLE XVIII. Ionization energies for some eighteen-electron atoms.

18 e Atom	Z	r_1 (a_o) a	r_3 (a_o) b	r_{10} (a_o) c	r_{12} (a_o) d	r_{18} (a_o) e	Theoretical Ionization Energies f (eV)	Experimental Ionization Energies g (eV)	Relative Error h
Ar	18	0.05599	0.23839	0.28878	0.78276	0.86680	15.69651	15.75962	0.0040
K^+	19	0.05302	0.22503	0.26884	0.71450	0.85215	31.9330	31.63	-0.0096
Ca^{2+}	20	0.05035	0.21308	0.25149	0.65725	0.82478	49.4886	50.9131	0.0280
Sc^{3+}	21	0.04794	0.20235	0.23625	0.60857	0.76196	71.4251	73.4894	0.0281
Ti^{4+}	22	0.04574	0.19264	0.22276	0.56666	0.70013	97.1660	99.30	0.0215
V^{5+}	23	0.04374	0.18383	0.21074	0.53022	0.64511	126.5449	128.13	0.0124
Cr^{6+}	24	0.04191	0.17579	0.19995	0.49822	0.59718	159.4836	160.18	0.0043
Mn^{7+}	25	0.04022	0.16842	0.19022	0.46990	0.55552	195.9359	194.5	-0.0074
Fe^{8+}	26	0.03867	0.16165	0.18140	0.44466	0.51915	235.8711	233.6	-0.0097
Co^{9+}	27	0.03723	0.15540	0.17336	0.42201	0.48720	279.2670	275.4	-0.0140
Ni^{10+}	28	0.03589	0.14961	0.16601	0.40158	0.45894	326.1070	321.0	-0.0159
Cu^{11+}	29	0.03465	0.14424	0.15926	0.38305	0.43379	376.3783	369	-0.0200
Zn^{12+}	30	0.03349	0.13925	0.15304	0.36617	0.41127	430.0704	419.7	-0.0247

a Radius of the paired 1s inner electrons of eighteen-electron atoms from Eq. (10.51) (Eq. (60)).

b Radius of the paired 2s inner electrons of eighteen-electron atoms from Eq. (10.62) (Eq. (60)).

c Radius of the three sets of paired 2p inner electrons of eighteen-electron atoms from Eq. (10.212)) (Eq. (64)).

d Radius of the paired 3s inner electrons of eighteen-electron atoms from Eq. (10.255)) (Eq. (60)).

e Radius of the three sets of paired 3p outer electrons of eighteen-electron atoms from Eq. (10.399) (Eq. (67)) for $Z > 18$ and Eq. (10.386) for Ar.

f Calculated ionization energies of eighteen-electron atoms given by the electric energy (Eq. (10.400)) (Eq. (61)).

g From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

h (Experimental-theoretical)/experimental.

TABLE XIX. Ionization energies for some nineteen-electron atoms.

19 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	r_{12} (a_o) [d]	r_{18} (a_o) [e]	r_{19} (a_o) [f]	Theoretical Ionization Energies [g] (eV)	Experimental Ionization Energies [h] (eV)	Relative Error [i]
K	19	0.053 02	0.225 03	0.268 84	0.714 50	0.852 15	3.145 15	4.32596	4.34066	0.0034
Ca^+	20	0.050 35	0.213 08	0.251 49	0.657 25	0.824 78	2.400 60	11.3354	11.87172	0.0452
Sc^{2+}	21	0.047 94	0.202 35	0.236 25	0.608 57	0.761 96	1.652 61	24.6988	24.75666	0.0023
Ti^{3+}	22	0.045 74	0.192 64	0.222 76	0.566 66	0.700 13	1.299 98	41.8647	43.2672	0.0324
V^{4+}	23	0.043 74	0.183 83	0.210 74	0.530 22	0.645 11	1.082 45	62.8474	65.2817	0.0373
Cr^{5+}	24	0.041 91	0.175 79	0.199 95	0.498 22	0.597 18	0.931 56	87.6329	90.6349	0.0331
Mn^{6+}	25	0.040 22	0.168 42	0.190 22	0.469 90	0.555 52	0.819 57	116.2076	119.203	0.0251
Fe^{7+}	26	0.038 67	0.161 65	0.181 40	0.444 66	0.519 15	0.732 67	148.5612	151.06	0.0165
Co^{8+}	27	0.037 23	0.155 40	0.173 36	0.422 01	0.487 20	0.663 03	184.6863	186.13	0.0078
Ni^{9+}	28	0.035 89	0.149 61	0.166 01	0.401 58	0.458 94	0.605 84	224.5772	224.6	0.0001
Cu^{10+}	29	0.034 65	0.144 24	0.159 26	0.383 05	0.433 79	0.557 97	268.2300	265.3	-0.0110
Zn^{11+}	30	0.033 49	0.139 25	0.153 04	0.366 17	0.411 27	0.517 26	315.6418	310.8	-0.0156

[a] Radius of the paired 1s inner electrons of nineteen-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the paired 2s inner electrons of nineteen-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the three sets of paired 2p inner electrons of nineteen-electron atoms from Eq. (10.212)) (Eq. (64)).

[d] Radius of the paired 3s inner electrons of nineteen-electron atoms from Eq. (10.255)) (Eq. (60)).

[e] Radius of the three sets of paired 3p inner electrons of nineteen-electron atoms from Eq. (10.399) (Eq. (67)).

[f] Radius of the unpaired 4s outer electron of nineteen-electron atoms from Eq. (10.425) (Eq. (60)) for $Z > 19$ and Eq. (10.414) for K.

[g] Calculated ionization energies of nineteen-electron atoms given by the electric energy (Eq. (10.426)) (Eq. (61)).

[h] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[i] (Experimental-theoretical)/experimental.

TABLE XX. Ionization energies for some twenty-electron atoms.

20 e Atom	Z	r_1 (a_o) [a]	r_3 (a_o) [b]	r_{10} (a_o) [c]	r_{12} (a_o) [d]	r_{18} (a_o) [e]	r_{20} (a_o) [f]	Theoretical Ionization Energies [g] (eV)	Experimental Ionization Energies [h] (eV)	Relative Error [i]
Ca	20	0.05035	0.21308	0.25149	0.65725	0.82478	2.23009	6.10101	6.11316	0.0020
Sc^+	21	0.04794	0.20235	0.23625	0.60857	0.76196	2.04869	13.2824	12.79967	-0.0377
Ti^{2+}	22	0.04574	0.19264	0.22276	0.56666	0.70013	1.48579	27.4719	27.4917	0.0007
V^{3+}	23	0.04374	0.18383	0.21074	0.53022	0.64511	1.19100	45.6956	46.709	0.0217
Cr^{4+}	24	0.04191	0.17579	0.19995	0.49822	0.59718	1.00220	67.8794	69.46	0.0228
Mn^{5+}	25	0.04022	0.16842	0.19022	0.46990	0.55552	0.86867	93.9766	95.6	0.0170
Fe^{6+}	26	0.03867	0.16165	0.18140	0.44466	0.51915	0.76834	123.9571	124.98	0.0082
Co^{7+}	27	0.03723	0.15540	0.17336	0.42201	0.48720	0.68977	157.8012	157.8	0.0000
Ni^{8+}	28	0.03589	0.14961	0.16601	0.40158	0.45894	0.62637	195.4954	193	-0.0129
Cu^{9+}	29	0.03465	0.14424	0.15926	0.38305	0.43379	0.57401	237.0301	232	-0.0217
Zn^{10+}	30	0.03349	0.13925	0.15304	0.36617	0.41127	0.52997	282.3982	274	-0.0307

[a] Radius of the paired 1s inner electrons of twenty-electron atoms from Eq. (10.51) (Eq. (60)).

[b] Radius of the paired 2s inner electrons of twenty-electron atoms from Eq. (10.62) (Eq. (60)).

[c] Radius of the three sets of paired 2p inner electrons of twenty-electron atoms from Eq. (10.212)) (Eq. (64)).

[d] Radius of the paired 3s inner electrons of twenty-electron atoms from Eq. (10.255)) (Eq. (60)).

[e] Radius of the three sets of paired 3p inner electrons of twenty-electron atoms from Eq. (10.399) (Eq. (67)).

[f] Radius of the paired 4s outer electrons of twenty-electron atoms from Eq. (10.445) (Eq. (60)) for $Z > 20$ and Eq. (10.436) for Ca.

[g] Calculated ionization energies of twenty-electron atoms given by the electric energy (Eq. (10.446)) (Eq. (61)).

[h] From theoretical calculations, interpolation of isoelectronic and spectral series, and experimental data [24-25].

[i] (Experimental-theoretical)/experimental.

GENERAL EQUATION FOR THE IONIZATION ENERGIES OF ATOMS HAVING AN OUTER S-SHELL

The derivation of the radii and energies of the 1s, 2s, 3s, and 4s electrons is given in the One-Electron Atom, the Two-Electron Atom, the Three-Electron Atoms, the Four-Electron Atoms, the Eleven-Electron Atoms, the Twelve-Electron Atoms, the Nineteen-Electron Atoms, and the Twenty-Electron Atoms sections of Ref. [4]. *(Reference to equations of the form Eq. (1.number), Eq. (7.number), and Eq. (10.number) will refer to the corresponding equations of Ref. [4].)* The general equation for the radii of s electrons is given by

$$r_n = \frac{\dfrac{a_0\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)} \pm a_0\sqrt{\left(\dfrac{\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]Er_m\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}}}{2} \qquad (60)$$

r_m *in units of* a_0

where Z is the nuclear charge, n is the number of electrons, r_m is the radius of the proceeding filled shell(s) given by Eq. (60) for the preceding s shell(s), Eq. (64) for the 2p shell, and Eq. (69) for the 3p shell, the parameter A given in TABLE XXI corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic}$, (Eq. (10.11)), the parameter B given in TABLE XXI corresponds to the paramagnetic force, $\mathbf{F}_{mag\,2}$ (Eq. (10.55)), the parameter C given in TABLE XXI corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic\,3}$, (Eq. (10.221)), the parameter D given in TABLE XXI corresponds to the paramagnetic force, $\mathbf{F}_{mag}$, (Eq. (7.15)), and the parameter E given in TABLE XXI corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic\,2}$, due to a relativistic effect with an electric field for $r > r_n$ (Eqs. (10.35), (10.229), and (10.418)). The positive root of Eq. (60) must be taken in order that $r_n > 0$. The radii of several n-electron atoms having an outer s shell are given in TABLES I-IV, XI-XII, XIX and XX.

The ionization energy for atoms having an outer s-shell are given by the negative of the electric energy, $E(electric)$, (Eq. (10.102) with the radii, r_n, given by Eq. (60) and Eq. (10.447)):

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n} \quad (61)$$

except that minor corrections due to the magnetic energy must be included in cases wherein the s electron does not couple to p electrons as given in Eqs. (7.28), (7.47), (10.25), (10.48), (10.66), and (10.68). Since the relativistic corrections were small except for one, two, and three-electron atoms, the nonrelativistic ionization energies for experimentally measured n-electron, s-filling atoms are given in most cases by Eqs. (60) and (61). The ionization energies of several n-electron atoms having an outer s shell are given in TABLES I-IV, XI-XII, XIX and XX.

TABLE XXI. Summary of the parameters of atoms filling the 1s, 2s, 3s, and 4s orbitals.

Atom Type	Electron Configuration	Ground State Term [a]	Orbital Arrangement of s Electrons (s state)	Diamag. Force Factor A [b]	Paramag. Force Factor B [c]	Diamag. Force Factor C [d]	Paramag. Force Factor D [e]	Diamag. Force Factor E [f]
Neutral 1 e Atom *H*	$1s^1$	$^2S_{1/2}$	↑ 1s	0	0	0	0	0
Neutral 2 e Atom *He*	$1s^2$	$^1S_0$	↑↓ 1s	0	0	0	1	0
Neutral 3 e Atom *Li*	$2s^1$	$^2S_{1/2}$	↑ 2s	1	0	0	0	0
Neutral 4 e Atom *Be*	$2s^2$	$^1S_0$	↑↓ 2s	1	0	0	1	0
Neutral 11 e Atom *Na*	$1s^2 2s^2 2p^6 3s^1$	$^2S_{1/2}$	↑ 3s	1	0	8	0	0
Neutral 12 e Atom *Mg*	$1s^2 2s^2 2p^6 3s^2$	$^1S_0$	↑↓ 3s	1	3	12	1	0
Neutral 19 e Atom *K*	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	↑ 4s	2	0	12	0	0
Neutral 20 e Atom *Ca*	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	$^1S_0$	↑↓ 4s	1	3	24	1	0
1 e Ion	$1s^1$	$^2S_{1/2}$	↑ 1s	0	0	0	0	0
2 e Ion	$1s^2$	$^1S_0$	↑↓ 1s	0	0	0	1	0
3 e Ion	$2s^1$	$^2S_{1/2}$	↑ 2s	1	0	0	0	1
4 e Ion	$2s^2$	$^1S_0$	↑↓ 2s	1	0	0	1	1
11 e Ion	$1s^2 2s^2 2p^6 3s^1$	$^2S_{1/2}$	↑ 3s	1	4	8	0	$1+\frac{\sqrt{2}}{2}$

12 e Ion	$1s^2 2s^2 2p^6 3s^2$	$^1S_0$	$\underline{\uparrow\downarrow}$ 3s	1	6	0	0	$1+\frac{\sqrt{2}}{2}$
19 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	$\underline{\uparrow}$ 4s	3	0	24	0	$2-\sqrt{2}$
20 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	$^1S_0$	$\underline{\uparrow\downarrow}$ 4s	2	0	24	0	$2-\sqrt{2}$

[a] The theoretical ground state terms match those given by NIST [26].
[b] Eq. (10.11).
[c] Eq. (10.55).
[d] Eq. (10.221).
[e] Eq. (7.15).
[f] Eqs. (10.35), (10.229), and (10.418).

GENERAL EQUATION FOR THE IONIZATION ENERGIES OF FIVE THROUGH TEN-ELECTRON ATOMS

The derivation of the radii and energies of the 2p electrons is given in the Five through Eight-Electron Atoms sections of Ref. [4]. Using the forces given by Eqs. (58) (Eq. (10.70)), (10.82-10.84), (10.89), (10.93), and the radii r_3 given by Eq. (10.62) (from Eq. (60)), the radii of the 2p electrons of all five through ten-electron atoms may be solved exactly. The electric energy given by Eq. (61) (Eq. (10.102)) gives the corresponding exact ionization energies. A summary of the parameters of the equations that determine the exact radii and ionization energies of all five through ten-electron atoms is given in TABLE XXII.

TABLE XXII. Summary of the parameters of five through ten-electron atoms.

Atom Type	Electron Configuration	Ground State Term [a]	Orbital Arrangement of 2p Electrons (2p state)	Diamagnetic Force Factor A [b]	Paramagnetic Force Factor B [c]
Neutral 5 e Atom B	$1s^2 2s^2 2p^1$	$^2P^0_{1/2}$	$\underline{\uparrow}\ \ \underline{\ \ }\ \ \underline{\ \ }$ 1 0 -1	2	0
Neutral 6 e Atom C	$1s^2 2s^2 2p^2$	$^3P_0$	$\underline{\uparrow}\ \ \underline{\uparrow}\ \ \underline{\ \ }$ 1 0 -1	$\frac{2}{3}$	0
Neutral 7 e Atom N	$1s^2 2s^2 2p^3$	$^4S^0_{3/2}$	$\underline{\uparrow}\ \ \underline{\uparrow}\ \ \underline{\uparrow}$ 1 0 -1	$\frac{1}{3}$	1
Neutral 8 e Atom O	$1s^2 2s^2 2p^4$	$^3P_2$	$\underline{\uparrow\downarrow}\ \ \underline{\uparrow}\ \ \underline{\uparrow}$ 1 0 -1	1	2
Neutral 9 e Atom F	$1s^2 2s^2 2p^5$	$^2P^0_{3/2}$	$\underline{\uparrow\downarrow}\ \ \underline{\uparrow\downarrow}\ \ \underline{\uparrow}$ 1 0 -1	$\frac{2}{3}$	3
Neutral 10 e Atom Ne	$1s^2 2s^2 2p^6$	$^1S_0$	$\underline{\uparrow\downarrow}\ \ \underline{\uparrow\downarrow}\ \ \underline{\uparrow\downarrow}$ 1 0 -1	0	3
5 e Ion	$1s^2 2s^2 2p^1$	$^2P^0_{1/2}$	$\underline{\uparrow}\ \ \underline{\ \ }\ \ \underline{\ \ }$ 1 0 -1	$\frac{5}{3}$	1
6 e Ion	$1s^2 2s^2 2p^2$	$^3P_0$	$\underline{\uparrow}\ \ \underline{\uparrow}\ \ \underline{\ \ }$ 1 0 -1	$\frac{5}{3}$	4
7 e Ion	$1s^2 2s^2 2p^3$	$^4S^0_{3/2}$	$\underline{\uparrow}\ \ \underline{\uparrow}\ \ \underline{\uparrow}$ 1 0 -1	$\frac{5}{3}$	6
8 e Ion	$1s^2 2s^2 2p^4$	$^3P_2$	$\underline{\uparrow\downarrow}\ \ \underline{\uparrow}\ \ \underline{\uparrow}$ 1 0 -1	$\frac{5}{3}$	6
9 e Ion	$1s^2 2s^2 2p^5$	$^2P^0_{3/2}$	$\underline{\uparrow\downarrow}\ \ \underline{\uparrow\downarrow}\ \ \underline{\uparrow}$ 1 0 -1	$\frac{5}{3}$	9
10 e Ion	$1s^2 2s^2 2p^6$	$^1S_0$	$\underline{\uparrow\downarrow}\ \ \underline{\uparrow\downarrow}\ \ \underline{\uparrow\downarrow}$ 1 0 -1	$\frac{5}{3}$	12

[a] The theoretical ground state terms match those given by NIST [26].

[b] Eq. (10.82).

[c] Eqs. (10.83-10.84) and (10.89).

$\mathbf{F}_{ele}$ and $\mathbf{F}_{diamagnetic\,2}$ given by Eqs. (58) (Eq. (10.70)) and (10.93), respectively, are of the same form for all atoms with the appropriate nuclear charges and atomic radii. $\mathbf{F}_{diamagnetic}$ given by Eq. (10.82) and $\mathbf{F}_{mag\,2}$ given by Eqs. (10.83-10.84) and (10.89) are of the same form with the appropriate factors that depend on the electron configuration wherein the electron configuration given in TABLE XXII must be a minimum of energy.

For each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^{n-4}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by Eqs. (7.19) and (10.51) (from Eq. (60)):

$$r_1 = r_2 = a_o \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right] \tag{62}$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by Eq. (10.62) (from Eq. (60)):

$$r_4 = r_3 = \frac{\left(\frac{a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)}{(Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1}} \pm a_o \sqrt{\frac{\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)^2}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2} + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1 10 \sqrt{\frac{3}{4}}}{(Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1}}} \right)}{2} \tag{63}$$

r_1 *in units of* a_o

where r_1 is given by Eq. (62), and $n-4$ electrons in an orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_3}\right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_3}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}\right)r_3\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_3}\right)}}}{2} \quad (64)$$

r_3 *in units of* a_0

where r_3 is given by Eq. (63), the parameter A given in TABLE XXII corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic}$, (Eq. (10.82)), and the parameter B given in TABLE XXII corresponds to the paramagnetic force, $\mathbf{F}_{mag\,2}$ (Eqs. (10.83-10.84) and (10.89)). The positive root of Eq. (64) must be taken in order that $r_n > 0$. The radii of several n-electron atoms are given in TABLES V-X.

The ionization energy for the boron atom is given by Eq. (10.104). The ionization energies for the n-electron atoms are given by the negative of the electric energy, $E(electric)$, (Eq. (61) with the radii, r_n, given by Eq. (64)). Since the relativistic corrections were small, the nonrelativistic ionization energies for experimentally measured n-electron atoms are given by Eqs. (61) and (64) in TABLES V-X.

GENERAL EQUATION FOR THE IONIZATION ENERGIES OF THIRTEEN THROUGH EIGHTEEN-ELECTRON ATOMS

The derivation of the radii and energies of the 3p electrons is given in the Thirteen through Eighteen-Electron Atoms sections of Ref. [4]. Using the forces given by Eqs. (58) (Eq.(10.257)), (10.258-10.264), (10.268), and the radii r_{12} given by Eq. (10.255) (from Eq. (60)), the radii of the 3p electrons of all thirteen through eighteen-electron atoms may be solved exactly. The electric energy given by Eq. (61) (Eq. (10.102)) gives the corresponding exact ionization energies. A summary of the parameters of the equations that determine the exact radii and ionization energies of all thirteen through eighteen-electron atoms is given in TABLES XIII-XVIII.

$\mathbf{F}_{ele}$ and $\mathbf{F}_{diamagnetic\,2}$ given by Eqs. (58) (Eq. (10.257)) and (10.268), respectively, are of the same form for all atoms with the appropriate nuclear charges

and atomic radii. $\mathbf{F}_{diamagnetic}$ given by Eq. (10.258) and $\mathbf{F}_{mag\,2}$ given by Eqs. (10.259-10.264) are of the same form with the appropriate factors that depend on the electron configuration given in TABLE XXIII wherein the electron configuration must be a minimum of energy.

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TABLE XXIII. Summary of the parameters of thirteen through eighteen-electron atoms.

Atom Type	Electron Configuration	Ground State Term [a]	Orbital Arrangement of 3p Electrons (3p state)	Diamagnetic Force Factor A [b]	Paramagnetic Force Factor B [c]
Neutral 13 e Atom *Al*	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P^0_{1/2}$	↑ ___ ___ (1, 0, -1)	$\frac{11}{3}$	0
Neutral 14 e Atom *Si*	$1s^2 2s^2 2p^6 3s^2 3p^2$	$^3P_0$	↑ ↑ ___ (1, 0, -1)	$\frac{7}{3}$	0
Neutral 15 e Atom *P*	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S^0_{3/2}$	↑ ↑ ↑ (1, 0, -1)	$\frac{5}{3}$	2
Neutral 16 e Atom *S*	$1s^2 2s^2 2p^6 3s^2 3p^4$	$^3P_2$	↑↓ ↑ ↑ (1, 0, -1)	$\frac{4}{3}$	1
Neutral 17 e Atom *Cl*	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P^0_{3/2}$	↑↓ ↑↓ ↑ (1, 0, -1)	$\frac{2}{3}$	2
Neutral 18 e Atom *Ar*	$1s^2 2s^2 2p^6 3s^2 3p^6$	$^1S_0$	↑↓ ↑↓ ↑↓ (1, 0, -1)	$\frac{1}{3}$	4
13 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P^0_{1/2}$	↑ ___ ___ (1, 0, -1)	$\frac{5}{3}$	12
14 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^2$	$^3P_0$	↑ ↑ ___ (1, 0, -1)	$\frac{1}{3}$	16
15 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S^0_{3/2}$	↑ ↑ ↑ (1, 0, -1)	0	24
16 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^4$	$^3P_2$	↑↓ ↑ ↑ (1, 0, -1)	$\frac{1}{3}$	24
17 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P^0_{3/2}$	↑↓ ↑↓ ↑ (1, 0, -1)	$\frac{2}{3}$	32
18 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6$	$^1S_0$	↑↓ ↑↓ ↑↓ (1, 0, -1)	0	40

[a] The theoretical ground state terms match those given by NIST [26].

b Eq. (10.258).
c Eqs. (10.260-10.264).

For each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^6 3s^2 3p^{n-12}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by Eq. (7.19) and (10.51) (from Eq. (60)):

$$r_1 = r_2 = a_o \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right] \tag{65}$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by Eq. (10.62) (from Eq. (60)):

$$r_4 = r_3 = \frac{\left(\frac{a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)} \pm a_o \sqrt{\frac{\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)^2}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2} + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1 10 \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)}} \right)}{2} \tag{66}$$

r_1 *in units of* a_o

where r_1 is given by Eq. (65), three sets of paired indistinguishable electrons in an orbitsphere with radius r_{10} given by Eq. (64) (Eq. (10.212)):

$$r_{10} = \frac{\frac{a_0}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-10}{Z-9}\right]\left(1-\frac{\sqrt{2}}{2}\right)r_3\right)}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)}}}{2} \quad (67)$$

r_3 *in units of* a_0

where r_3 is given by Eq. (66) (Eqs. (10.62) and (10.402)), two indistinguishable spin-paired electrons in an orbitsphere with radius r_{12} given by Eq. (10.255) (from Eq. (60)):

$$r_{12} = \frac{\frac{a_0}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-12}{Z-11}\right]\left(1+\frac{\sqrt{2}}{2}\right)r_{10}\right)}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}}}{2} \quad (68)$$

r_{10} *in units of* a_0

where r_{10} is given by Eq. (67) (Eq. (10.212)), and $n-12$ electrons in a 3p orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)r_{12}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}}}{2} \quad (69)$$

r_{12} *in units of* a_0

where r_{12} is given by Eq. (68) (Eqs. (10.255) and (10.404)), the parameter A given

in TABLE XXIII corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic}$, (Eq. (10.258)), and the parameter B given in TABLE XXIII corresponds to the paramagnetic force, $\mathbf{F}_{mag\,2}$ (Eqs. (10.260-10.264)). The positive root of Eq. (69) must be taken in order that $r_n > 0$. The radii of several n-electron 3p atoms are given in TABLES XIII-XVIII.

The ionization energy for the aluminum atom is given by Eq. (10.227). The ionization energies for the n-electron 3p atoms are given by the negative of the electric energy, $E(electric)$, (Eq. (61) with the radii, r_n, given by Eq. (69)). Since the relativistic corrections were small, the nonrelativistic ionization energies for experimentally measured n-electron 3p atoms are given by Eqs. (61) and (69) in TABLES XIII-XVIII.

Systems

Embodiments of the system for performing computing and rendering of the nature atomic and atomic-ionic electrons using the physical solutions may comprise a general purpose computer. Such a general purpose computer may have any number of basic configurations. For example, such a general purpose computer may comprise a central processing unit (CPU), one or more specialized processors, system memory, a mass storage device such as a magnetic disk, an optical disk, or other storage device, an input means such as a keyboard or mouse, a display device, and a printer or other output device. A system implementing the present invention can also comprise a special purpose computer or other hardware system and all should be included within its scope.

The display can be static or dynamic such that spin and angular motion with corresponding momenta can be displayed in an embodiment. The displayed information is useful to anticipate reactivity and physical properties. The insight into the nature of atomic and atomic-ionic electrons can permit the solution and display of other atoms and atomic ions and provide utility to anticipate their reactivity and physical properties. Furthermore, the displayed information is useful in teaching environments to teach students the properties of electrons.

Embodiments within the scope of the present invention also include computer program products comprising computer readable medium having embodied therein program code means. Such computer readable media can be any available media which can be accessed by a general purpose or special purpose computer. By way of example, and not limitation, such computer readable media can comprise RAM,

ROM, EPROM, CD ROM, DVD or other optical disk storage, magnetic disk storage or other magnetic storage devices, or any other medium which can embody the desired program code means and which can be accessed by a general purpose or special purpose computer. Combinations of the above should also be included within the scope of computer readable media. Program code means comprises, for example, executable instructions and data which cause a general purpose computer or special purpose computer to perform a certain function of a group of functions.

A specific example of the rendering of the electron of atomic hydrogen using Mathematica and computed on a PC is shown in FIGURE 1. The algorithm used was

To generate a spherical shell:

SphericalPlot3D[1,{q,0,p},{f,0,2p},Boxed®False,Axes®False];. The rendering can be viewed from different perspectives. A specific example of the rendering of atomic hydrogen using Mathematica and computed on a PC is shown in FIGURE 1. The algorithm used was

To generate the picture of the electron and proton:

Electron=SphericalPlot3D[1,{q,0,p},{f,0,2p-p/2},Boxed®False,Axes®False];
Proton=Show[Graphics3D[{Blue,PointSize[0.03],Point[{0,0,0}]}],Boxed®False];
Show[Electron,Proton];

Specific examples of the rendering of the spherical-and-time-harmonic-electron-charge-density functions using Mathematica and computed on a PC are shown in FIGURE 3. The algorithm used was

To generate L1MO:

L1MOcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

L1MO=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta] Sin[phi],Cos[theta],L1MOcolors[theta,phi,1+Cos[theta]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-2.030,3.494}];

To generate L1MX:

L1MXcolors[theta_, phi_, det_] = Which[det < 0.1333, RGBColor[1.000, 0.070, 0.079],det < .2666, RGBColor[1.000, 0.369, 0.067],det < .4, RGBColor[1.000, 0.681, 0.049],det < .5333, RGBColor[0.984, 1.000, 0.051], det < .6666, RGBColor[0.673, 1.000, 0.058], det < .8, RGBColor[0.364, 1.000, 0.055],det < .9333, RGBColor[0.071, 1.000, 0.060], det < 1.066, RGBColor[0.085, 1.000, 0.388],det < 1.2, RGBColor[0.070, 1.000, 0.678], det < 1.333, RGBColor[0.070, 1.000, 1.000],det < 1.466, RGBColor[0.067, 0.698, 1.000], det < 1.6, RGBColor[0.075, 0.401, 1.000],det < 1.733, RGBColor[0.067, 0.082, 1.000], det < 1.866, RGBColor[0.326, 0.056, 1.000],det <= 2, RGBColor[0.674, 0.079, 1.000]];

L1MX=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta] Sin[phi],Cos[theta],L1MXcolors[theta,phi,1+Sin[theta] Cos[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-2.030,3.494}];

To generate L1MY:

L1MYcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

L1MY=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta] Sin[phi],Cos[theta],L1MYcolors[theta,phi,1+Sin[theta] Sin[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoint

s®{20,20}];

To generate L2MO:

```
L2MOcolors[theta_, phi_, det_] = Which[det < 0.2, RGBColor[1.000, 0.070,
0.079],det < .4, RGBColor[1.000, 0.369, 0.067],det < .6, RGBColor[1.000, 0.681,
0.049],det < .8, RGBColor[0.984, 1.000, 0.051],det < 1, RGBColor[0.673, 1.000,
0.058],det < 1.2, RGBColor[0.364, 1.000, 0.055],det < 1.4, RGBColor[0.071, 1.000,
0.060],det < 1.6, RGBColor[0.085, 1.000, 0.388],det < 1.8, RGBColor[0.070, 1.000,
0.678],det < 2, RGBColor[0.070, 1.000, 1.000],det < 2.2, RGBColor[0.067, 0.698,
1.000],det < 2.4, RGBColor[0.075, 0.401, 1.000],det < 2.6, RGBColor[0.067, 0.082,
1.000],det < 2.8, RGBColor[0.326, 0.056, 1.000],det <= 3, RGBColor[0.674, 0.079,
1.000]];

L2MO=ParametricPlot3D[{Sin[theta] Cos[phi], Sin[theta] Sin[phi], Cos[theta],
      L2MOcolors[theta, phi, 3Cos[theta] Cos[theta]]},
      {theta, 0, Pi}, {phi, 0, 2Pi},
      Boxed -> False, Axes -> False, Lighting -> False,
      PlotPoints-> {20, 20},
      ViewPoint->{-0.273, -2.030, 3.494}];
```

To generate L2MF:

```
L2MFcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<
.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333
,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGB
Color[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBC
olor[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[
0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.07
5,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,
0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

L2MF=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]
Sin[phi],Cos[theta],L2MFcolors[theta,phi,1+.72618 Sin[theta] Cos[phi] 5 Cos[theta]
Cos[theta]-.72618 Sin[theta]
Cos[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoin
```

ts®{20,20},ViewPoint®{-0.273,-2.030,2.494}];

To generate L2MX2Y2:

L2MX2Y2colors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

L2MX2Y2=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta] Sin[phi],Cos[theta],L2MX2Y2colors[theta,phi,1+Sin[theta] Sin[theta] Cos[2 phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-2.030,3.494}];

To generate L2MXY:

L2MXYcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta] Sin[phi],Cos[theta],L2MXYcolors[theta,phi,1+Sin[theta] Sin[theta] Sin[2 phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-2.030,3.494}];

The present invention may be embodied in other specific forms without departing from the spirit or essential attributes thereof and, accordingly, reference should be made to the appended claims, rather than to the foregoing specification,

as indicating the scope of the invention.

The following list of references are incorporated by reference in their entirety and referred to throughout this application by use of brackets.

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I Claim:

1. A system of computing and rendering the nature of bound atomic and atomic ionic electrons from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration, comprising:

processing means for processing and solving the equations for charge, mass, and current density functions of electron(s) in a selected atom or ion, wherein the equations are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration; and

a display in communication with the processing means for displaying the current and charge density representation of the electron(s) of the selected atom or ion.

2. The system of claim 1, wherein the display is at least one of visual or graphical media.

3. The system of claim 1, wherein the display is at least one of static or dynamic.

4. The system of claim 3, wherein the processing means is constructed and arranged so that at least one of spin and orbital angular motion can be displayed.

5. The system of claim 1, wherein the processing means is constructed and arranged so that the displayed information can be used to model reactivity and physical properties.

6. The system of claim 1, wherein the processing means is constructed and arranged so that the displayed information can be used to model other atoms and atomic ions and provide utility to anticipate their reactivity and physical properties.

7. The system of claim 1, wherein the processing means is a general purpose computer.

8. The system of claim 7, wherein the general purpose computer comprises a central processing unit (CPU), one or more specialized processors, system memory, a mass storage device such as a magnetic disk, an optical disk, or other storage

device, an input means such as a keyboard or mouse, a display device, and a printer or other output device.

9. The system of claim 1, wherein the processing means comprises a special purpose computer or other hardware system.

10. The system of claim 1, further comprising computer program products.

11. The system of claim 1, further comprising computer readable media having embodied therein program code means in communication with the processing means.

12. The system of claim 11, wherein the computer readable media is any available media that can be accessed by a general purpose or special purpose computer.

13. The system of claim 12, wherein the computer readable media comprises at least one of RAM, ROM, EPROM, CD ROM, DVD or other optical disk storage, magnetic disk storage or other magnetic storage devices, or any other medium that can embody a desired program code means and that can be accessed by a general purpose or special purpose computer.

14. The system of claim 13, wherein the program code means comprises executable instructions and data which cause a general purpose computer or special purpose computer to perform a certain function of a group of functions.

15. The system of claim 14, wherein the program code is Mathematica programmed with an algorithm based on the physical solutions.

16. The system of claim 15, wherein the algorithm for the rendering of the electron of atomic hydrogen using Mathematica is

```
SphericalPlot3D[1,{q,0,p},{f,0,2p},Boxed®False,Axes®False];
```

and the algorithm for the rendering of atomic hydrogen using Mathematica and computed on a PC is

```
Electron=SphericalPlot3D[1,{q,0,p},{f,0,2p-p/2},Boxed®False,Axes®False];
Proton=Show[Graphics3D[{Blue,PointSize[0.03],Point[{0,0,0}]}],Boxed®False];
Show[Electron,Proton];
```

17. The system of claim 15, wherein the algorithm for the rendering of the spherical-and-time-harmonic-electron-charge-density functions using Mathematica are

To generate L1MO;

```
L1MOcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det
<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.533
3,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RG
BColor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGB
Color[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColo
r[0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.0
75,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.32
6,0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

L1MO=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]
Sin[phi],Cos[theta],L1MOcolors[theta,phi,1+Cos[theta]]},{theta,0,Pi},{phi,0,2Pi},Boxe
d®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-
2.030,3.494}];
```

To generate L1MX;

```
L1MXcolors[theta_, phi_, det_] = Which[det < 0.1333, RGBColor[1.000, 0.070,
0.079],det < .2666, RGBColor[1.000, 0.369, 0.067],det < .4, RGBColor[1.000, 0.681,
0.049],det < .5333, RGBColor[0.984, 1.000, 0.051],    det < .6666, RGBColor[0.673,
1.000, 0.058],          det < .8, RGBColor[0.364, 1.000, 0.055],det < .9333,
RGBColor[0.071, 1.000, 0.060],   det < 1.066, RGBColor[0.085, 1.000, 0.388],det <
1.2, RGBColor[0.070, 1.000, 0.678],       det < 1.333, RGBColor[0.070, 1.000,
1.000],det < 1.466, RGBColor[0.067, 0.698, 1.000],    det < 1.6, RGBColor[0.075,
0.401, 1.000],det < 1.733, RGBColor[0.067, 0.082, 1.000],    det < 1.866,
RGBColor[0.326, 0.056, 1.000],det <= 2, RGBColor[0.674, 0.079, 1.000]];

L1MX=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]
Sin[phi],Cos[theta],L1MXcolors[theta,phi,1+Sin[theta]
Cos[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoint
s®{20,20},ViewPoint®{-0.273,-2.030,3.494}];
```

To generate L1MY;

```
L1MYcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<
.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333
,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGB
Color[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBC
olor[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[
0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.07
5,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,
0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

L1MY=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]
Sin[phi],Cos[theta],L1MYcolors[theta,phi,1+Sin[theta]
Sin[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoint
s®{20,20}];
```

To generate L2MO;

```
L2MOcolors[theta_, phi_, det_] = Which[det < 0.2, RGBColor[1.000, 0.070,
0.079],det < .4, RGBColor[1.000, 0.369, 0.067],det < .6, RGBColor[1.000, 0.681,
0.049],det < .8, RGBColor[0.984, 1.000, 0.051],det < 1, RGBColor[0.673, 1.000,
0.058],det < 1.2, RGBColor[0.364, 1.000, 0.055],det < 1.4, RGBColor[0.071, 1.000,
0.060],det < 1.6, RGBColor[0.085, 1.000, 0.388],det < 1.8, RGBColor[0.070, 1.000,
0.678],det < 2, RGBColor[0.070, 1.000, 1.000],det < 2.2, RGBColor[0.067, 0.698,
1.000],det < 2.4, RGBColor[0.075, 0.401, 1.000],det < 2.6, RGBColor[0.067, 0.082,
1.000],det < 2.8, RGBColor[0.326, 0.056, 1.000],det <= 3, RGBColor[0.674, 0.079,
1.000]];

L2MO=ParametricPlot3D[{Sin[theta] Cos[phi], Sin[theta] Sin[phi], Cos[theta],
      L2MOcolors[theta, phi, 3Cos[theta] Cos[theta]]},
      {theta, 0, Pi}, {phi, 0, 2Pi},
      Boxed -> False, Axes -> False, Lighting -> False,
      PlotPoints-> {20, 20},
      ViewPoint->{-0.273, -2.030, 3.494}];
```

To generate L2MF;

L2MFcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

L2MF=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta] Sin[phi],Cos[theta],L2MFcolors[theta,phi,1+.72618 Sin[theta] Cos[phi] 5 Cos[theta] Cos[theta]-.72618 Sin[theta] Cos[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-2.030,2.494}];

To generate L2MX2Y2;

L2MX2Y2colors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

L2MX2Y2=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta] Sin[phi],Cos[theta],L2MX2Y2colors[theta,phi,1+Sin[theta] Sin[theta] Cos[2 phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-2.030,3.494}];

To generate L2MXY;

L2MXYcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RG

BColor[0.085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£2,RGBColor[0.674,0.079,1.000]];

ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta] Sin[phi],Cos[theta],L2MXYcolors[theta,phi,1+Sin[theta] Sin[theta] Sin[2 phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-2.030,3.494}].

18. The system of claim 1 wherein the physical, Maxwellian solutions of the charge, mass, and current density functions of atoms and atomic ions comprises a solution of the classical wave equation $\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]\rho(r,\theta,\phi,t) = 0$.

19. The system of claim 18, wherein the time, radial, and angular solutions of the wave equation are separable.

20. The system of claim 18, wherein the boundary constraint of the wave equation solution is nonradiation according to Maxwell's equations.

21. The system of claim 20, wherein a radial function that satisfies the boundary condition is a radial delta function

$$f(r) = \frac{1}{r^2}\delta(r - r_n).$$

22. The system of claim 21, wherein the boundary condition is met for a time harmonic function when the relationship between an allowed radius and the electron wavelength is given by

$$2\pi r_n = \lambda_n,$$

$$\omega = \frac{\hbar}{m_e r^2}, \text{ and}$$

$$v = \frac{\hbar}{m_e r}$$

where ω is the angular velocity of each point on the electron surface, v is the velocity of each point on the electron surface, and r is the radius of the electron.

23. The system of claim 22, wherein the spin function is given by the uniform

function $Y_0^0(\phi,\theta)$ comprising angular momentum components of $\mathbf{L}_{xy}=\frac{\hbar}{4}$ and $\mathbf{L}_z=\frac{\hbar}{2}$.

24. The system of claim 23, wherein the atomic and atomic ionic charge and current density functions of bound electrons are described by a charge-density (mass-density) function which is the product of a radial delta function, two angular functions (spherical harmonic functions), and a time harmonic function:

$$\rho(r,\theta,\phi,t)=f(r)A(\theta,\phi,t)=\frac{1}{r^2}\delta(r-r_n)A(\theta,\phi,t);\qquad A(\theta,\phi,t)=Y(\theta,\phi)k(t)$$

wherein the spherical harmonic functions correspond to a traveling charge density wave confined to the spherical shell which gives rise to the phenomenon of orbital angular momentum.

25. The system of claim 24, wherein based on the radial solution, the angular charge and current-density functions of the electron, $A(\theta,\phi,t)$, must be a solution of the wave equation in two dimensions (plus time),

$$\left[\nabla^2-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]A(\theta,\phi,t)=0$$

where $\rho(r,\theta,\phi,t)=f(r)A(\theta,\phi,t)=\frac{1}{r^2}\delta(r-r_n)A(\theta,\phi,t)$ and $A(\theta,\phi,t)=Y(\theta,\phi)k(t)$

$$\left[\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)_{r,\phi}+\frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)_{r,\theta}-\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]A(\theta,\phi,t)=0$$

where v is the linear velocity of the electron.

26. The system of claim 25, wherein the charge-density functions including the time-function factor are

$\ell = 0$

$$\rho(r,\theta,\phi,t)=\frac{e}{8\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi)+Y_\ell^m(\theta,\phi)\right]$$

$\ell \neq 0$

$$\rho(r,\theta,\phi,t)=\frac{e}{4\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi)+\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\}\right]$$

where $Y_\ell^m(\theta,\phi)$ are the spherical harmonic functions that spin about the z-axis with angular frequency ω_n with $Y_0^0(\theta,\phi)$ the constant function

$\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\} = P_\ell^m(\cos\theta)\cos\left(m\phi + \omega_n' t\right)$ where to keep the form of the spherical harmonic as a traveling wave about the z-axis, $\omega_n' = m\omega_n$.

27. The system of claim 26, wherein the spin and angular moment of inertia, I, angular momentum, L, and energy, E, for quantum number ℓ are given by

$\ell = 0$

$$I_z = I_{spin} = \frac{m_e r_n^{\,2}}{2}$$

$$L_z = I\omega \mathbf{i}_z = \pm\frac{\hbar}{2}$$

$$E_{rotational} = E_{rotational,\,spin} = \frac{1}{2}\left[I_{spin}\left(\frac{\hbar}{m_e r_n^{\,2}}\right)^2\right] = \frac{1}{2}\left[\frac{m_e r_n^{\,2}}{2}\left(\frac{\hbar}{m_e r_n^{\,2}}\right)^2\right] = \frac{1}{4}\left[\frac{\hbar^2}{2I_{spin}}\right]$$

$\ell \neq 0$

$$I_{orbital} = m_e r_n^{\,2}\left[\frac{\ell(\ell+1)}{\ell^2+\ell+1}\right]^{\frac{1}{2}}$$

$$L_z = m\hbar$$

$$L_{z\,total} = L_{z\,spin} + L_{z\,orbital}$$

$$E_{rotational,\,orbital} = \frac{\hbar^2}{2I}\left[\frac{\ell(\ell+1)}{\ell^2+2\ell+1}\right]$$

$$T = \frac{\hbar^2}{2m_e r_n^2}$$

$$\left\langle E_{rotational,\,orbital}\right\rangle = 0.$$

28. The system of claim 1, wherein the force balance equation for one-electron atoms and ions is

$$\frac{m_e}{4\pi r_1^2}\frac{v_1^2}{r_1} = \frac{e}{4\pi r_1^2}\frac{Ze}{4\pi\varepsilon_o r_1^2} - \frac{1}{4\pi r_1^2}\frac{\hbar^2}{m_p r_n^3}$$

$$r_1 = \frac{a_H}{Z}$$

where a_H is the radius of the hydrogen atom.

29. The system of claim 28, wherein from Maxwell's equations, the potential energy V, kinetic energy T, electric energy or binding energy E_{ele} are

$$V = \frac{-Ze^2}{4\pi\varepsilon_o r_1} = \frac{-Z^2e^2}{4\pi\varepsilon_o a_H} = -Z^2\,X\,4.3675\,X\,10^{-18}\,J = -Z^2\,X\,27.2\;eV$$

$$T = \frac{Z^2e^2}{8\pi\varepsilon_o a_H} = Z^2\,X\,13.59\;eV$$

$$T = E_{ele} = -\frac{1}{2}\varepsilon_o\int_\infty^{r_1}\mathbf{E}^2 dv \quad \text{where } \mathbf{E} = -\frac{Ze}{4\pi\varepsilon_o r^2}$$

$$E_{ele} = -\frac{Z^2 e^2}{8\pi\varepsilon_o a_H} = -Z^2 X 2.1786 X 10^{-18} J = -Z^2 X 13.598 eV.$$

30. The system of claim 1, wherein the force balance equation solution of two electron atoms is a central force balance equation with the nonradiation condition given by

$$\frac{m_e}{4\pi r_2^2}\frac{v_2^2}{r_2} = \frac{e}{4\pi r_2^2}\frac{(Z-1)e}{4\pi\varepsilon_o r_2^2} + \frac{1}{4\pi r_2^2}\frac{\hbar^2}{Z m_e r_2^3}\sqrt{s(s+1)}$$

which gives the radius of both electrons as

$$r_2 = r_1 = a_0\left(\frac{1}{Z-1} - \frac{\sqrt{s(s+1)}}{Z(Z-1)}\right); s = \frac{1}{2}.$$

31. The system of claim 30, wherein the ionization energy for helium, which has no electric field beyond r_1 is given by

$$Ionization\ Energy(He) = -E(electric) + E(magnetic)$$

where,

$$E(electric) = -\frac{(Z-1)e^2}{8\pi\varepsilon_o r_1}$$

$$E(magnetic) = \frac{2\pi\mu_0 e^2\hbar^2}{m_e^2 r_1^3}$$

For $3 \le Z$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy.$$

32. The system of claim 1, wherein the electrons of multielectron atoms all exist as orbitspheres of discrete radii which are given by r_n of the radial Dirac delta function, $\delta(r - r_n)$.

33. The system of claim 32, wherein electron orbitspheres may be spin paired or unpaired depending on the force balance which applies to each electron wherein the electron configuration is a minimum of energy.

34. The system of claim 33, wherein the minimum energy configurations are given by solutions to Laplace's equation.

35. The system of claim 34, wherein the electrons of an atom with the same principal and ℓ quantum numbers align parallel until each of the m_ℓ levels are occupied, and then pairing occurs until each of the m_ℓ levels contain paired

electrons.

36. The system of claim 35, wherein the electron configuration for one through twenty-electron atoms that achieves an energy minimum is: 1s < 2s < 2p < 3s < 3p < 4s.

37. The system of claim 36, wherein the corresponding force balance of the central centrifical, Coulombic, paramagnetic, magnetic, and diamagnetic forces for an electron configuration was derived for each n-electron atom that was solved for the radius of each electron.

38. The system of claim 37, wherein the central Coulombic force is that of a point charge at the origin since the electron charge-density functions are spherically symmetrical with a time dependence that is nonradiative.

39. The system of claim 38, wherein the ionization energies are obtained using the calculated radii in the determination of the Coulombic and any magnetic energies.

40. The system of claim 39, wherein the general equation for the radii of s electrons is given by

$$r_n = \frac{\dfrac{a_0\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)} \pm a_0\sqrt{\left(\dfrac{\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]Er_m\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}}}{2}$$

r_m *in units of* a_0

where positive root must be taken in order that $r_n > 0$;

Z is the nuclear charge, n is the number of electrons,

r_m is the radius of the proceeding filled shell(s) given by

$$r_n = \frac{\dfrac{a_0\left(1+(C-D)\dfrac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\dfrac{A}{8}-\dfrac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_m}\right)} \pm a_0\sqrt{\left(\dfrac{\left(1+(C-D)\dfrac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\dfrac{A}{8}-\dfrac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_m}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\dfrac{Z-n}{Z-(n-1)}\right]Er_m\right)}{\left((Z-(n-1))-\left(\dfrac{A}{8}-\dfrac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_m}\right)}}}{2}$$

r_m *in units of* a_0

for the preceding s shell(s);

$$r_n = \frac{\dfrac{a_0}{\left((Z-(n-1))-\left(\dfrac{A}{8}-\dfrac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_3}\right)} \pm a_0\sqrt{\left(\dfrac{1}{\left((Z-(n-1))-\left(\dfrac{A}{8}-\dfrac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_3}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\dfrac{Z-n}{Z-(n-1)}\right]\left(1-\dfrac{\sqrt{2}}{2}\right)r_3\right)}{\left((Z-(n-1))-\left(\dfrac{A}{8}-\dfrac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_3}\right)}}}{2}$$

r_3 *in units of* a_0

for the 2p shell, and

$$r_n = \frac{\dfrac{a_0}{\left((Z-(n-1))-\left(\dfrac{A}{8}-\dfrac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_{12}}\right)} \pm a_0\sqrt{\left(\dfrac{1}{\left((Z-(n-1))-\left(\dfrac{A}{8}-\dfrac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_{12}}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\dfrac{Z-n}{Z-(n-1)}\right]\left(1-\dfrac{\sqrt{2}}{2}+\dfrac{1}{2}\right)r_{12}\right)}{\left((Z-(n-1))-\left(\dfrac{A}{8}-\dfrac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_{12}}\right)}}}{2}$$

r_{12} *in units of* a_0

for the 3p shell;

the parameter A corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic}$:

$$\mathbf{F}_{diamagnetic} = -\frac{\hbar^2}{4m_e r_3^2 r_1}\sqrt{s(s+1)}\mathbf{i}_r;$$

the parameter B corresponds to the paramagnetic force, $\mathbf{F}_{mag\,2}$:

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_1 r_4^2}\sqrt{s(s+1)}\mathbf{i}_r;$$

the parameter C corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic\,3}$:

$$\mathbf{F}_{diamagnetic\,3} = -\frac{1}{Z}\frac{8\hbar^2}{m_e r_{11}^3}\sqrt{s(s+1)}\mathbf{i}_r;$$

the parameter D corresponds to the paramagnetic force, $\mathbf{F}_{mag}$:

$$\mathbf{F}_{mag} = \frac{1}{4\pi r_2^2}\frac{1}{Z}\frac{\hbar^2}{m_e r^3}\sqrt{s(s+1)}, \text{ and}$$

the parameter E corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic\,2}$, due to a relativistic effect with an electric field for $r > r_n$:

$$\mathbf{F}_{diamagnetic\,2} = -\left\lfloor\frac{Z-3}{Z-2}\right\rfloor\frac{r_1\hbar^2}{m_e r_3^4}10\sqrt{3/4}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\,2} = -\left\lfloor\frac{Z-11}{Z-10}\right\rfloor\left(1+\frac{\sqrt{2}}{2}\right)\frac{r_{10}\hbar^2}{m_e r_{11}^4}10\sqrt{s(s+1)}\mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{diamagnetic\,2} = -\left\lfloor\frac{Z-n}{Z-(n-1)}\right\rfloor\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\frac{r_{18}\hbar^2}{m_e r_n^4}10\sqrt{s(s+1)}\mathbf{i}_r.$$

wherein the parameters of atoms filling the 1s, 2s, 3s, and 4s orbitals are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of s Electrons (s state)	Diamag. Force Factor *A*	Paramag. Force Factor *B*	Diamag. Force Factor *C*	Para mag. Force Factor *D*	Diamag. Force Factor *E*
Neutral 1 e Atom *H*	$1s^1$	$^2S_{1/2}$	 1s	0	0	0	0	0
Neutral 2 e Atom *He*	$1s^2$	$^1S_0$	 1s	0	0	0	1	0
Neutral 3 e Atom *Li*	$2s^1$	$^2S_{1/2}$	 2s	1	0	0	0	0
Neutral 4 e Atom *Be*	$2s^2$	$^1S_0$	 2s	1	0	0	1	0
Neutral 11 e Atom *Na*	$1s^2 2s^2 2p^6 3s^1$	$^2S_{1/2}$	 3s	1	0	8	0	0
Neutral 12 e Atom *Mg*	$1s^2 2s^2 2p^6 3s^2$	$^1S_0$	 3s	1	3	12	1	0
Neutral 19 e Atom *K*	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	4s	2	0	12	0	0
Neutral 20 e Atom *Ca*	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	$^1S_0$	4s	1	3	24	1	0
1 e Ion	$1s^1$	$^2S_{1/2}$	 1s	0	0	0	0	0
2 e Ion	$1s^2$	$^1S_0$	 1s	0	0	0	1	0
3 e Ion	$2s^1$	$^2S_{1/2}$	 2s	1	0	0	0	1
4 e Ion	$2s^2$	$^1S_0$	2s	1	0	0	1	1

11 e Ion	$1s^2 2s^2 2p^6 3s^1$	$^2S_{1/2}$	$\underline{\uparrow}$ 3s	1	4	8	0	$1+\frac{\sqrt{2}}{2}$
12 e Ion	$1s^2 2s^2 2p^6 3s^2$	$^1S_0$	$\underline{\uparrow\downarrow}$ 3s	1	6	0	0	$1+\frac{\sqrt{2}}{2}$
19 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	$\underline{\uparrow}$ 4s	3	0	24	0	$2-\sqrt{2}$
20 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	$^1S_0$	$\underline{\uparrow\downarrow}$ 4s	2	0	24	0	$2-\sqrt{2}$

.

41. The system of claim 40, with the radii, r_n, wherein the ionization energy for atoms having an outer s-shell are given by the negative of the electric energy, $E(electric)$, given by:

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}$$

except that minor corrections due to the magnetic energy must be included in cases wherein the s electron does not couple to p electrons as given by

$$Ionization\ Energy(He) = -E(electric) + E(magnetic)\left(1-\frac{1}{2}\left(\left(\frac{2}{3}\cos\frac{\pi}{3}\right)^2+\alpha\right)\right)$$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z}Magnetic\ Energy$$

$$E(ionization;\ Li) = \frac{(Z-2)e^2}{8\pi\varepsilon_o r_3} + \Delta E_{mag}$$

$$= 5.3178\ eV + 0.0860\ eV = 5.4038\ eV$$

$$E(Ionization) = E(Electric) + E_T$$

$$E(ionization;\ Be) = \frac{(Z-3)e^2}{8\pi\varepsilon_o r_4} + \frac{2\pi\mu_0 e^2\hbar^2}{m_e^2 r_4^3} + \Delta E_{mag}$$, and

$$= 8.9216\ eV + 0.03226\ eV + 0.33040\ eV = 9.28430\ eV$$

$$E(Ionization) = -Electric\ Energy - \frac{1}{Z}Magnetic\ Energy - E_T.$$

42. The system of claim 41, wherein the radii and energies of the 2p electrons are solved using the forces given by

$$\mathbf{F}_{ele} = \frac{(Z-n)e^2}{4\pi\varepsilon_o r_n^2}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!}\frac{\hbar^2}{4m_e r_n^2 r_3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\,2} = -\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}\right)\frac{r_3\hbar^2}{m_e r_n^4}10\sqrt{s(s+1)}\mathbf{i}_r,$$

and the radii r_3 are given by

$$r_4 = r_3 = \frac{\left(\frac{a_0\left(1-\frac{\sqrt{\frac{3}{4}}}{Z}\right)}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)} \pm a_o\sqrt{\frac{\left(1-\frac{\sqrt{\frac{3}{4}}}{Z}\right)^2}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)^2}+4\frac{\left[\frac{Z-3}{Z-2}\right]r_1 10\sqrt{\frac{3}{4}}}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)}}\right)}{2}$$

r_1 *in units of* a_o

43. The system of claim 42, wherein the electric energy given by

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}$$

gives the corresponding ionization energies.

44. The system of claim 43, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^{n-4}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_o \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right];$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(\frac{a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)} \right.}{2} \left. \pm a_o \sqrt{\frac{\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)^2}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2} + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1 10 \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)}} \right)$$

r_1 *in units of* a_o

and $n-4$ electrons in an orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)^2 + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)}}}{2};$$

r_3 *in units of* a_0

the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic}$:

$$\mathbf{F}_{diamagnetic} = -\sum_{m} \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4 m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r;$$

and the parameter B corresponds to the paramagnetic force, $\mathbf{F}_{mag\,2}$:

$$\mathbf{F}_{mag\,2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r,$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

wherein the parameters of five through ten-electron atoms are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of 2p Electrons (2p state)	Diamagnetic Force Factor *A*	Paramagnetic Force Factor *B*
Neutral 5 e Atom *B*	$1s^2 2s^2 2p^1$	$^2P^0_{1/2}$	$\underline{\uparrow}\ \underline{\ }\ \underline{\ }$ 1 0 -1	2	0
Neutral 6 e Atom *C*	$1s^2 2s^2 2p^2$	$^3P_0$	$\underline{\uparrow}\ \underline{\uparrow}\ \underline{\ }$ 1 0 -1	$\frac{2}{3}$	0
Neutral 7 e Atom *N*	$1s^2 2s^2 2p^3$	$^4S^0_{3/2}$	$\underline{\uparrow}\ \underline{\uparrow}\ \underline{\uparrow}$ 1 0 -1	$\frac{1}{3}$	1
Neutral 8 e Atom *O*	$1s^2 2s^2 2p^4$	$^3P_2$	$\underline{\uparrow\downarrow}\ \underline{\uparrow}\ \underline{\uparrow}$ 1 0 -1	1	2
Neutral 9 e Atom *F*	$1s^2 2s^2 2p^5$	$^2P^0_{3/2}$	$\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow}$ 1 0 -1	$\frac{2}{3}$	3
Neutral 10 e Atom *Ne*	$1s^2 2s^2 2p^6$	$^1S_0$	$\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}$ 1 0 -1	0	3
5 e Ion	$1s^2 2s^2 2p^1$	$^2P^0_{1/2}$	$\underline{\uparrow}\ \underline{\ }\ \underline{\ }$ 1 0 -1	$\frac{5}{3}$	1
6 e Ion	$1s^2 2s^2 2p^2$	$^3P_0$	$\underline{\uparrow}\ \underline{\uparrow}\ \underline{\ }$ 1 0 -1	$\frac{5}{3}$	4
7 e Ion	$1s^2 2s^2 2p^3$	$^4S^0_{3/2}$	$\underline{\uparrow}\ \underline{\uparrow}\ \underline{\uparrow}$ 1 0 -1	$\frac{5}{3}$	6
8 e Ion	$1s^2 2s^2 2p^4$	$^3P_2$	$\underline{\uparrow\downarrow}\ \underline{\uparrow}\ \underline{\uparrow}$ 1 0 -1	$\frac{5}{3}$	6
9 e Ion	$1s^2 2s^2 2p^5$	$^2P^0_{3/2}$	$\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow}$ 1 0 -1	$\frac{5}{3}$	9
10 e Ion	$1s^2 2s^2 2p^6$	$^1S_0$	$\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}$ 1 0 -1	$\frac{5}{3}$	12

.

45. The system of claim 44, wherein the ionization energy for the boron atom is given by

$$E(ionization;\ B) = \frac{(Z-4)e^2}{8\pi\varepsilon_o r_5} + \Delta E_{mag}$$
$$= 8.147170901\ eV + 0.15548501\ eV = 8.30265592\ eV$$

46. The system of claim 44, wherein the ionization energies for the n-electron atoms having the radii, r_n ,are given by the negative of the electric energy, $E(electric)$, given by

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}.$$

47. The system of claim 1, wherein the radii of the 3p electrons are given using the forces given by

$$\mathbf{F}_{ele} = \frac{(Z-n)e^2}{4\pi\varepsilon_o r_n^2}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!}\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\left(\frac{2}{3}+\frac{2}{3}+\frac{1}{3}\right)\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r = -\left(\frac{5}{3}\right)\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = (4+4+4)\frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r = \frac{1}{Z}\frac{12\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{8\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

and the radii r_{12} are given by

$$r_{12} = \frac{\dfrac{a_0}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)} \pm a_0 \sqrt{\left(\dfrac{1}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left\lceil\frac{Z-12}{Z-11}\right\rceil\left(1+\frac{\sqrt{2}}{2}\right)r_{10}\right)}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}}}{2}$$

r_{10} *in units of* a_0

48. The system of claim 47, wherein the ionization energies are given by electric energy given by:

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}.$$

49. The system of claim 1, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^6 3s^2 3p^{n-12}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_o\left\lceil\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)}\right\rceil$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(\frac{a_0\left(1-\frac{\sqrt{\frac{3}{4}}}{Z}\right)}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)} \pm a_o \sqrt{\frac{\left(1-\frac{\sqrt{\frac{3}{4}}}{Z}\right)^2}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)^2} + 4\frac{\left[\frac{Z-3}{Z-2}\right] r_1 10\sqrt{\frac{3}{4}}}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)}} \right)}{2}$$

r_1 *in units of* a_o

three sets of paired indistinguishable electrons in an orbitsphere with radius r_{10} given by:

$$r_{10} = \frac{\frac{a_0}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-10}{Z-9}\right]\left(1-\frac{\sqrt{2}}{2}\right)r_3\right)}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)}}}{2}$$

r_3 *in units of* a_0

two indistinguishable spin-paired electrons in an orbitsphere with radius r_{12} given by:

$$r_{12} = \frac{\frac{a_0}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-12}{Z-11}\right]\left(1+\frac{\sqrt{2}}{2}\right)r_{10}\right)}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}}}{2}$$

r_{10} *in units of* a_0

and $n-12$ electrons in a 3p orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)r_{12}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}}}{2}$$

r_{12} *in units of* a_0

where the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic}$:

$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!}\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$, and the parameter B corresponds to the paramagnetic force, $\mathbf{F}_{mag\,2}$:

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = (4+4+4)\frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r = \frac{1}{Z}\frac{12\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{8\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

wherein the parameters of thirteen through eighteen-electron atoms are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of 3p Electrons (3p state)	Diamagnetic Force Factor *A*	Paramagnetic Force Factor *B*
Neutral 13 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P^0_{1/2}$	↑ — — (1, 0, -1)	11/3	0
Al Neutral 14 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^2$	$^3P_0$	↑ ↑ — (1, 0, -1)	7/3	0
Si Neutral 15 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S^0_{3/2}$	↑ ↑ ↑ (1, 0, -1)	5/3	2
P Neutral 16 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^4$	$^3P_2$	↑↓ ↑ ↑ (1, 0, -1)	4/3	1
S Neutral 17 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P^0_{3/2}$	↑↓ ↑↓ ↑ (1, 0, -1)	2/3	2
Cl Neutral 18 e Atom	$1s^2 2s^2 2p^6 3s^2 3p^6$	$^1S_0$	↑↓ ↑↓ ↑↓ (1, 0, -1)	1/3	4
Ar 13 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P^0_{1/2}$	↑ — — (1, 0, -1)	5/3	12
14 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^2$	$^3P_0$	↑ ↑ — (1, 0, -1)	1/3	16
15 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S^0_{3/2}$	↑ ↑ ↑ (1, 0, -1)	0	24
16 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^4$	$^3P_2$	↑↓ ↑ ↑ (1, 0, -1)	1/3	24
17 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P^0_{3/2}$	↑↓ ↑↓ ↑ (1, 0, -1)	2/3	32
18 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6$	$^1S_0$	↑↓ ↑↓ ↑↓ (1, 0, -1)	0	40

50. The system of claim 49, wherein the ionization energies for the n-electron 3p atoms are given by electric energy given by:

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}.$$

51. The system of claim 50, wherein the ionization energy for the aluminum atom is given by

$$E(ionization;\ Al) = \frac{(Z-12)e^2}{8\pi\varepsilon_o r_{13}} + \Delta E_{mag}$$
$$= 5.95270\ eV + 0.031315\ eV = 5.98402\ eV$$

52. A system of computing the nature of bound atomic and atomic ionic electrons from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration, comprising:

processing means for processing and solving the equations for charge, mass, and current density functions of electron(s) in selected atoms or ions, wherein the equations are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration; and

output means for outputting the solutions of the charge, mass, and current density functions of the atoms and atomic ions.

53. A method comprising the steps of;

a.) inputting electron functions that are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration;

b.) inputting a trial electron configuration;

c.) inputting the corresponding centrifugal, Coulombic, diamagnetic and paramagnetic forces,

d.) forming the force balance equation comprising the centrifugal force equal to the sum of the Coulombic, diamagnetic and paramagnetic forces;

e.) solving the force balance equation for the electron radii;

f.) calculating the energy of the electrons using the radii and the corresponding electric and magnetic energies;

g.) repeating Steps a-f for all possible electron configurations, and

h.) outputting the lowest energy configuration and the corresponding electron radii for that configuration.

54. The method of claim 53, wherein the output is rendered using the electron functions.

55. The method of claim 54, wherein the electron functions are given by at least one of the group comprising:

$\ell = 0$

$$\rho(r,\theta,\phi,t) = \frac{e}{8\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi) + Y_\ell^m(\theta,\phi)\right]$$

$\ell \neq 0$

$$\rho(r,\theta,\phi,t) = \frac{e}{4\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi) + \mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\}\right]$$

where $Y_\ell^m(\theta,\phi)$ are the spherical harmonic functions that spin about the z-axis with angular frequency ω_n with $Y_0^0(\theta,\phi)$ the constant function. $\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\} = P_\ell^m(\cos\theta)\cos\left(m\phi + \omega_n' t\right)$ where to keep the form of the spherical harmonic as a traveling wave about the z-axis, $\omega_n' = m\omega_n$.

56. The method of claim 55, wherein the forces are given by at least one of the group comprising:

$$\mathbf{F}_{ele} = \frac{(Z-n)e^2}{4\pi\varepsilon_o r_n^2}\mathbf{i}_r$$

$$\mathbf{F}_{ele} = \frac{(Z-(n-1))e^2}{4\pi\varepsilon_o r_n^2}\mathbf{i}_r$$

$$\mathbf{F}_{mag} = \frac{1}{4\pi r_2^2}\frac{1}{Z}\frac{\hbar^2}{m_e r^3}\sqrt{s(s+1)}$$

$$\mathbf{F}_{diamagnetic} = -\frac{\hbar^2}{4m_e r_3^2 r_1}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!}\frac{\hbar^2}{4m_e r_n^2 r_3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!}\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\left(\frac{2}{3}+\frac{2}{3}+\frac{1}{3}\right)\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r = -\left(\frac{5}{3}\right)\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-3}{Z-2}\right]\frac{r_1\hbar^2}{m_e r_3^4}10\sqrt{3/4}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}\right)\frac{r_3\hbar^2}{m_e r_n^4}10\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-11}{Z-10}\right]\left(1+\frac{\sqrt{2}}{2}\right)\frac{r_{10}\hbar^2}{m_e r_{11}^4}10\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\frac{r_{18}\hbar^2}{m_e r_n^4}10\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 3} = -\frac{1}{Z}\frac{8\hbar^2}{m_e r_{11}^3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_1 r_4^2}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = (4+4+4)\frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r = \frac{1}{Z}\frac{12\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z}\frac{8\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

57. The method of claim 53, wherein the radii are given by at least one of the group comprising:

$$r_1 = r_2 = a_o\left[\frac{1}{Z-1}-\frac{\sqrt{\frac{3}{4}}}{Z(Z-1)}\right]$$

$$r_4 = r_3 = \frac{\dfrac{a_0\left(1-\dfrac{\sqrt{\frac{3}{4}}}{Z}\right)}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\dfrac{\sqrt{\frac{3}{4}}}{r_1}\right)} \pm a_o \sqrt{\dfrac{\left(1-\dfrac{\sqrt{\frac{3}{4}}}{Z}\right)^2}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\dfrac{\sqrt{\frac{3}{4}}}{r_1}\right)^2} + 4\dfrac{\left[\frac{Z-3}{Z-2}\right]r_1 10\sqrt{\frac{3}{4}}}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\dfrac{\sqrt{\frac{3}{4}}}{r_1}\right)}}}{2}$$

r_1 in units of a_o

$$r_n = \frac{\dfrac{a_0}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\dfrac{\sqrt{3}}{r_3}\right)} \pm a_0 \sqrt{\dfrac{\left(\dfrac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_3}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}\right)r_3\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_3}\right)}}}{2}$$

r_3 in units of a_0

$$r_{10} = \frac{\dfrac{a_0}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\dfrac{\sqrt{3}}{r_3}\right)} \pm a_0 \sqrt{\dfrac{\left(\dfrac{1}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-10}{Z-9}\right]\left(1-\frac{\sqrt{2}}{2}\right)r_3\right)}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)}}}{2}$$

r_3 in units of a_0

$$r_{11} = \frac{a_0\left(1+\dfrac{8}{Z}\sqrt{\frac{3}{4}}\right)}{(Z-10)-\dfrac{\sqrt{\frac{3}{4}}}{4r_{10}}}, \quad r_{10} \text{ in units of } a_0$$

$$r_{12} = \frac{\dfrac{a_0}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)} \pm a_0 \sqrt{\left(\dfrac{1}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-12}{Z-11}\right]\left(1+\frac{\sqrt{2}}{2}\right)r_{10}\right)}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}}}{2}$$

r_{10} *in units of* a_0

$$r_n = \frac{\dfrac{a_0}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)} \pm a_0 \sqrt{\left(\dfrac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)r_{12}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}}}{2}$$

r_{12} *in units of* a_0

$$r_n = \frac{\dfrac{a_0\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)} \pm a_0 \sqrt{\left(\dfrac{\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]Er_m\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}}}{2}$$

r_m *in units of* a_0

58. The method of claim 53, wherein the electric energy of each electron of radius r_n is given by at least one of the group comprising:

$$E(electric) = -\frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}$$

$$Ionization\ Energy(He) = -E(electric) + E(magnetic)\left(1-\frac{1}{2}\left(\left(\frac{2}{3}\cos\frac{\pi}{3}\right)^2+\alpha\right)\right)$$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy$$

$$E(Ionization) = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy - E_T$$

$$E(ionization;\ Li) = \frac{(Z-2)e^2}{8\pi\varepsilon_o r_3} + \Delta E_{mag}$$

$$= 5.3178\ eV + 0.0860\ eV = 5.4038\ eV$$

$$E(ionization;\ B) = \frac{(Z-4)e^2}{8\pi\varepsilon_o r_5} + \Delta E_{mag}$$

$$= 8.147170901\ eV + 0.15548501\ eV = 8.30265592\ eV$$

$$E(ionization;\ Be) = \frac{(Z-3)e^2}{8\pi\varepsilon_o r_4} + \frac{2\pi\mu_0 e^2\hbar^2}{m_e^2 r_4^3} + \Delta E_{mag}$$

$$= 8.9216\ eV + 0.03226\ eV + 0.33040\ eV = 9.28430\ eV$$

$$E(ionization;\ Na) = -Electric\ Energy = \frac{(Z-10)e^2}{8\pi\varepsilon_o r_{11}} = 5.12592\ eV$$

59. The method of claim 53, wherein the radii of s electrons are given by

$$r_n = \frac{\dfrac{a_0\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)} \pm a_0 \sqrt{\left(\dfrac{\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]Er_m\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}}}{2}$$

r_m *in units of* a_0

where positive root must be taken in order that $r_n > 0$;

Z is the nuclear charge, n is the number of electrons,

r_m is the radius of the proceeding filled shell(s) given by

$$r_n = \frac{\frac{a_0\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)} \pm a_0\sqrt{\left(\frac{\left(1+(C-D)\frac{\sqrt{3}}{2Z}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]Er_m\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_m}\right)}}}{2}$$

r_m *in units of* a_0

for the preceding s shell(s);

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_3}\right)} \pm a_0\sqrt{\left(\frac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_3}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}\right)r_3\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_3}\right)}}}{2}$$

r_3 *in units of* a_0

for the 2p shell, and

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)} \pm a_0\sqrt{\left(\frac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)r_{12}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}}}{2}$$

r_{12} *in units of* a_0

for the 3p shell;

the parameter A corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic}$:

$$\mathbf{F}_{diamagnetic} = -\frac{\hbar^2}{4m_e r_3^2 r_1}\sqrt{s(s+1)}\mathbf{i}_r;$$

the parameter B corresponds to the paramagnetic force, $\mathbf{F}_{mag\,2}$:

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_1 r_4^2}\sqrt{s(s+1)}\mathbf{i}_r;$$

the parameter C corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic\,3}$:

$$\mathbf{F}_{diamagnetic\,3} = -\frac{1}{Z}\frac{8\hbar^2}{m_e r_{11}^{\ 3}}\sqrt{s(s+1)}\mathbf{i}_r;$$

the parameter D corresponds to the paramagnetic force, $\mathbf{F}_{mag}$:

$$\mathbf{F}_{mag} = \frac{1}{4\pi r_2^{\ 2}}\frac{1}{Z}\frac{\hbar^2}{m_e r^3}\sqrt{s(s+1)}, \text{ and}$$

the parameter E corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic\,2}$, due to a relativistic effect with an electric field for $r > r_n$:

$$\mathbf{F}_{diamagnetic\,2} = -\left[\frac{Z-3}{Z-2}\right]\frac{r_1\hbar^2}{m_e r_3^4}10\sqrt{3/4}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\,2} = -\left[\frac{Z-11}{Z-10}\right]\left(1+\frac{\sqrt{2}}{2}\right)\frac{r_{10}\hbar^2}{m_e r_{11}^4}10\sqrt{s(s+1)}\mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{diamagnetic\,2} = -\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\frac{r_{18}\hbar^2}{m_e r_n^4}10\sqrt{s(s+1)}\mathbf{i}_r.$$

wherein the parameters of atoms filling the 1s, 2s, 3s, and 4s orbitals are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of s Electrons (s state)	Dia mag. Force Factor *A*	Para mag. Force Factor *B*	Dia mag. Force Factor *C*	Para mag. Force Factor *D*	Diamag. Force Factor *E*
Neutral 1 e Atom *H*	$1s^1$	$^2S_{1/2}$	↑ 1s	0	0	0	0	0
Neutral 2 e Atom *He*	$1s^2$	$^1S_0$	↑↓ 1s	0	0	0	1	0
Neutral 3 e Atom *Li*	$2s^1$	$^2S_{1/2}$	↑ 2s	1	0	0	0	0
Neutral 4 e Atom *Be*	$2s^2$	$^1S_0$	↑↓ 2s	1	0	0	1	0
Neutral 11 e Atom *Na*	$1s^2 2s^2 2p^6 3s^1$	$^2S_{1/2}$	↑ 3s	1	0	8	0	0
Neutral 12 e Atom *Mg*	$1s^2 2s^2 2p^6 3s^2$	$^1S_0$	↑↓ 3s	1	3	12	1	0
Neutral 19 e Atom *K*	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	↑ 4s	2	0	12	0	0
Neutral 20 e Atom *Ca*	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	$^1S_0$	↑↓ 4s	1	3	24	1	0
1 e Ion	$1s^1$	$^2S_{1/2}$	↑ 1s	0	0	0	0	0
2 e Ion	$1s^2$	$^1S_0$	↑↓ 1s	0	0	0	1	0
3 e Ion	$2s^1$	$^2S_{1/2}$	↑ 2s	1	0	0	0	1

4 e Ion	$2s^2$	$^1S_0$	$\underline{\uparrow\downarrow}$ 2s	1	0	0	1	1
11 e Ion	$1s^2 2s^2 2p^6 3s^1$	$^2S_{1/2}$	$\underline{\uparrow}$ 3s	1	4	8	0	$1+\frac{\sqrt{2}}{2}$
12 e Ion	$1s^2 2s^2 2p^6 3s^2$	$^1S_0$	$\underline{\uparrow\downarrow}$ 3s	1	6	0	0	$1+\frac{\sqrt{2}}{2}$
19 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	$\underline{\uparrow}$ 4s	3	0	24	0	$2-\sqrt{2}$
20 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	$^1S_0$	$\underline{\uparrow\downarrow}$ 4s	2	0	24	0	$2-\sqrt{2}$

.

60. The method of claim 59, with the radii, r_n, wherein the ionization energy for atoms having an outer s-shell are given by the negative of the electric energy, $E(electric)$, given by:

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}$$

except that minor corrections due to the magnetic energy must be included in cases wherein the s electron does not couple to p electrons as given by

$$Ionization\ Energy(He) = -E(electric) + E(magnetic)\left(1-\frac{1}{2}\left(\left(\frac{2}{3}\cos\frac{\pi}{3}\right)^2+\alpha\right)\right)$$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z}Magnetic\ Energy$$

$$E(ionization;\ Li) = \frac{(Z-2)e^2}{8\pi\varepsilon_o r_3} + \Delta E_{mag}$$

$$= 5.3178\ eV + 0.0860\ eV = 5.4038\ eV$$

$$E(Ionization) = E(Electric) + E_T$$

$$E(ionization;\ Be) = \frac{(Z-3)e^2}{8\pi\varepsilon_o r_4} + \frac{2\pi\mu_0 e^2\hbar^2}{m_e^2 r_4^3} + \Delta E_{mag}$$, and

$$= 8.9216\ eV + 0.03226\ eV + 0.33040\ eV = 9.28430\ eV$$

$$E(Ionization) = -Electric\ Energy - \frac{1}{Z}Magnetic\ Energy - E_T.$$

61. The method of claim 53, wherein the radii and energies of the 2p electrons are solved using the forces given by

$$\mathbf{F}_{ele} = \frac{(Z-n)e^2}{4\pi\varepsilon_o r_n^2}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_{m} \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\,2} = -\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}\right)\frac{r_3\hbar^2}{m_e r_n^4} 10\sqrt{s(s+1)}\mathbf{i}_r,$$

and the radii r_3 are given by

$$r_4 = r_3 = \frac{\left(\frac{a_0\left(1-\frac{\sqrt{\frac{3}{4}}}{Z}\right)}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)} \pm a_o\sqrt{\frac{\left(1-\frac{\sqrt{\frac{3}{4}}}{Z}\right)^2}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)^2} + 4\frac{\left[\frac{Z-3}{Z-2}\right]r_1 10\sqrt{\frac{3}{4}}}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)}}\right)}{2}$$

r_1 *in units of* a_o

62. The method of claim 61, wherein the electric energy given by

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}$$

gives the corresponding ionization energies.

63. The method of claim 53, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^{n-4}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_o \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right];$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(\frac{a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)} \pm a_o \sqrt{\frac{\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)^2}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2} + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1 10 \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)}} \right)}{2}$$

r_1 *in units of* a_o

and $n-4$ electrons in an orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)^2 + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)}}}{2};$$

r_3 *in units of* a_0

the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic}$:

$$\mathbf{F}_{diamagnetic} = -\sum_{m} \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)}\mathbf{i}_r;$$

and the parameter B corresponds to the paramagnetic force, $\mathbf{F}_{mag\,2}$:

$$\mathbf{F}_{mag\,2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)}\mathbf{i}_r,$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)}\mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)}\mathbf{i}_r.$$

wherein the parameters of five through ten-electron atoms are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of 2p Electrons (2p state)	Diamagnetic Force Factor A	Paramagnetic Force Factor B
Neutral 5 e Atom B	$1s^2 2s^2 2p^1$	$^2P^0_{1/2}$	$\underline{\uparrow}\ \underline{\ }\ \underline{\ }$ (1, 0, -1)	2	0
Neutral 6 e Atom C	$1s^2 2s^2 2p^2$	$^3P_0$	$\underline{\uparrow}\ \underline{\uparrow}\ \underline{\ }$ (1, 0, -1)	$\frac{2}{3}$	0
Neutral 7 e Atom N	$1s^2 2s^2 2p^3$	$^4S^0_{3/2}$	$\underline{\uparrow}\ \underline{\uparrow}\ \underline{\uparrow}$ (1, 0, -1)	$\frac{1}{3}$	1
Neutral 8 e Atom O	$1s^2 2s^2 2p^4$	$^3P_2$	$\underline{\uparrow\downarrow}\ \underline{\uparrow}\ \underline{\uparrow}$ (1, 0, -1)	1	2
Neutral 9 e Atom F	$1s^2 2s^2 2p^5$	$^2P^0_{3/2}$	$\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow}$ (1, 0, -1)	$\frac{2}{3}$	3
Neutral 10 e Atom Ne	$1s^2 2s^2 2p^6$	$^1S_0$	$\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}$ (1, 0, -1)	0	3
5 e Ion	$1s^2 2s^2 2p^1$	$^2P^0_{1/2}$	$\underline{\uparrow}\ \underline{\ }\ \underline{\ }$ (1, 0, -1)	$\frac{5}{3}$	1
6 e Ion	$1s^2 2s^2 2p^2$	$^3P_0$	$\underline{\uparrow}\ \underline{\uparrow}\ \underline{\ }$ (1, 0, -1)	$\frac{5}{3}$	4
7 e Ion	$1s^2 2s^2 2p^3$	$^4S^0_{3/2}$	$\underline{\uparrow}\ \underline{\uparrow}\ \underline{\uparrow}$ (1, 0, -1)	$\frac{5}{3}$	6
8 e Ion	$1s^2 2s^2 2p^4$	$^3P_2$	$\underline{\uparrow\downarrow}\ \underline{\uparrow}\ \underline{\uparrow}$ (1, 0, -1)	$\frac{5}{3}$	6
9 e Ion	$1s^2 2s^2 2p^5$	$^2P^0_{3/2}$	$\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow}$ (1, 0, -1)	$\frac{5}{3}$	9
10 e Ion	$1s^2 2s^2 2p^6$	$^1S_0$	$\underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}\ \underline{\uparrow\downarrow}$ (1, 0, -1)	$\frac{5}{3}$	12

64. The method of claim 63, wherein the ionization energy for the boron atom is given by

$$E(ionization;\ B) = \frac{(Z-4)e^2}{8\pi\varepsilon_o r_5} + \Delta E_{mag}$$
$$= 8.147170901\ eV + 0.15548501\ eV = 8.30265592\ eV$$

65. The method of claim 63, wherein the ionization energies for the n-electron atoms having the radii, r_n ,are given by the negative of the electric energy, $E(electric)$, given by

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}.$$

66. The method of claim 53, wherein the radii of the 3p electrons are given using the forces given by

$$\mathbf{F}_{ele} = \frac{(Z-n)e^2}{4\pi\varepsilon_o r_n^2}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!}\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\left(\frac{2}{3}+\frac{2}{3}+\frac{1}{3}\right)\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r = -\left(\frac{5}{3}\right)\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = (4+4+4)\frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r = \frac{1}{Z}\frac{12\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{8\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

and the radii r_{12} are given by

$$r_{12} = \frac{\frac{a_0}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)} \pm a_0\sqrt{\left(\frac{1}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}\right)^2 + \frac{20\sqrt{3}\left(\left\lceil\frac{Z-12}{Z-11}\right\rceil\left(1+\frac{\sqrt{2}}{2}\right)r_{10}\right)}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}}}{2}$$

r_{10} *in units of* a_0

67. The method of claim 66, wherein the ionization energies are given by electric energy given by:

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}.$$

68. The method of claim 53, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^6 3s^2 3p^{n-12}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_o\left\lceil\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)}\right\rceil$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(\frac{a_0\left(1-\frac{\sqrt{\frac{3}{4}}}{Z}\right)}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)} \pm a_o \sqrt{\frac{\left(1-\frac{\sqrt{\frac{3}{4}}}{Z}\right)^2}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)^2} + 4\frac{\left[\frac{Z-3}{Z-2}\right] r_1 10\sqrt{\frac{3}{4}}}{\left((Z-3)-\left(\frac{1}{4}-\frac{1}{Z}\right)\frac{\sqrt{\frac{3}{4}}}{r_1}\right)}} \right)}{2}$$

r_1 *in units of* a_o

three sets of paired indistinguishable electrons in an orbitsphere with radius r_{10} given by:

$$r_{10} = \frac{\frac{a_0}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)} \pm a_0 \sqrt{\frac{1}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)^2} + \frac{20\sqrt{3}\left(\left[\frac{Z-10}{Z-9}\right]\left(1-\frac{\sqrt{2}}{2}\right) r_3\right)}{\left((Z-9)-\left(\frac{5}{24}-\frac{6}{Z}\right)\frac{\sqrt{3}}{r_3}\right)}}}{2}$$

r_3 *in units of* a_0

two indistinguishable spin-paired electrons in an orbitsphere with radius r_{12} given by:

$$r_{12} = \frac{\dfrac{a_0}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)} \pm a_0 \sqrt{\left(\dfrac{1}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-12}{Z-11}\right]\left(1+\frac{\sqrt{2}}{2}\right)r_{10}\right)}{\left((Z-11)-\left(\frac{1}{8}-\frac{3}{Z}\right)\frac{\sqrt{3}}{r_{10}}\right)}}}{2}$$

r_{10} *in units of* a_0

and $n-12$ electrons in a 3p orbitsphere with radius r_n given by

$$r_n = \frac{\dfrac{a_0}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)} \pm a_0 \sqrt{\left(\dfrac{1}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}\right)^2 + \dfrac{20\sqrt{3}\left(\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)r_{12}\right)}{\left((Z-(n-1))-\left(\frac{A}{8}-\frac{B}{2Z}\right)\frac{\sqrt{3}}{r_{12}}\right)}}}{2}$$

r_{12} *in units of* a_0

where the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic}$:

$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!}\frac{\hbar^2}{4m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$, and the parameter B corresponds to the paramagnetic force, $\mathbf{F}_{mag\,2}$:

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = (4+4+4)\frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r = \frac{1}{Z}\frac{12\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{mag\,2} = \frac{1}{Z}\frac{8\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r,$$

wherein the parameters of thirteen to eighteen-electron atoms are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of 3p Electrons (3p state)	Diamagnetic Force Factor *A*	Paramagnetic Force Factor *B*
Neutral 13 e Atom *Al*	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P^0_{1/2}$	$\underline{\uparrow}_1 \quad \underline{\quad}_0 \quad \underline{\quad}_{-1}$	$\frac{11}{3}$	0
Neutral 14 e Atom *Si*	$1s^2 2s^2 2p^6 3s^2 3p^2$	$^3P_0$	$\underline{\uparrow}_1 \quad \underline{\uparrow}_0 \quad \underline{\quad}_{-1}$	$\frac{7}{3}$	0
Neutral 15 e Atom *P*	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S^0_{3/2}$	$\underline{\uparrow}_1 \quad \underline{\uparrow}_0 \quad \underline{\uparrow}_{-1}$	$\frac{5}{3}$	2
Neutral 16 e Atom *S*	$1s^2 2s^2 2p^6 3s^2 3p^4$	$^3P_2$	$\underline{\uparrow\downarrow}_1 \quad \underline{\uparrow}_0 \quad \underline{\uparrow}_{-1}$	$\frac{4}{3}$	1
Neutral 17 e Atom *Cl*	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P^0_{3/2}$	$\underline{\uparrow\downarrow}_1 \quad \underline{\uparrow\downarrow}_0 \quad \underline{\uparrow}_{-1}$	$\frac{2}{3}$	2
Neutral 18 e Atom *Ar*	$1s^2 2s^2 2p^6 3s^2 3p^6$	$^1S_0$	$\underline{\uparrow\downarrow}_1 \quad \underline{\uparrow\downarrow}_0 \quad \underline{\uparrow\downarrow}_{-1}$	$\frac{1}{3}$	4
13 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P^0_{1/2}$	$\underline{\uparrow}_1 \quad \underline{\quad}_0 \quad \underline{\quad}_{-1}$	$\frac{5}{3}$	12
14 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^2$	$^3P_0$	$\underline{\uparrow}_1 \quad \underline{\uparrow}_0 \quad \underline{\quad}_{-1}$	$\frac{1}{3}$	16
15 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S^0_{3/2}$	$\underline{\uparrow}_1 \quad \underline{\uparrow}_0 \quad \underline{\uparrow}_{-1}$	0	24
16 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^4$	$^3P_2$	$\underline{\uparrow\downarrow}_1 \quad \underline{\uparrow}_0 \quad \underline{\uparrow}_{-1}$	$\frac{1}{3}$	24
17 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P^0_{3/2}$	$\underline{\uparrow\downarrow}_1 \quad \underline{\uparrow\downarrow}_0 \quad \underline{\uparrow}_{-1}$	$\frac{2}{3}$	32
18 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6$	$^1S_0$	$\underline{\uparrow\downarrow}_1 \quad \underline{\uparrow\downarrow}_0 \quad \underline{\uparrow\downarrow}_{-1}$	0	40

69. The method of claim 68 wherein the ionization energies for the n-electron 3p atoms are given by electric energy given by:

$$E(Ionization) = -Electric\ Energy = \frac{(Z-(n-1))e^2}{8\pi\varepsilon_o r_n}.$$

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70. The method of claim 68 wherein the ionization energy for the aluminum atom is given by

$$E(ionization; Al) = \frac{(Z-12)e^2}{8\pi\varepsilon_o r_{13}} + \Delta E_{mag}$$
$$= 5.95270\ eV + 0.031315\ eV = 5.98402\ eV$$

Fig. 1

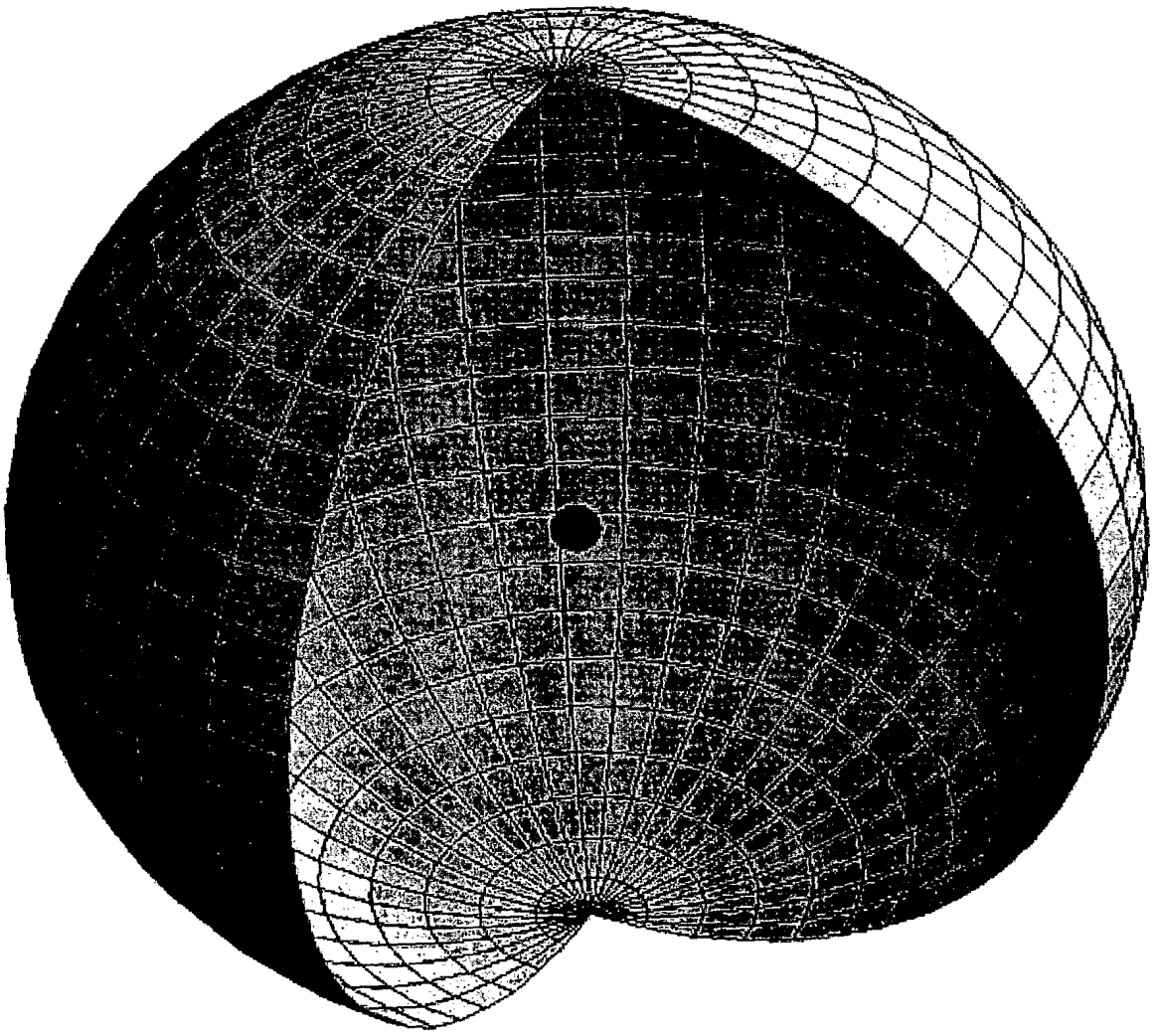


Fig. 2

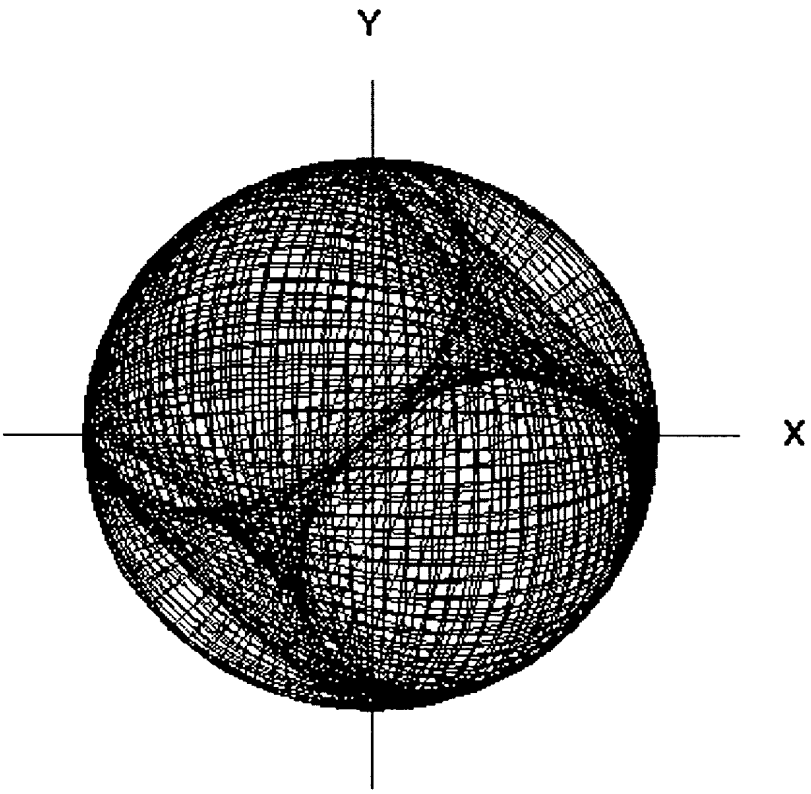
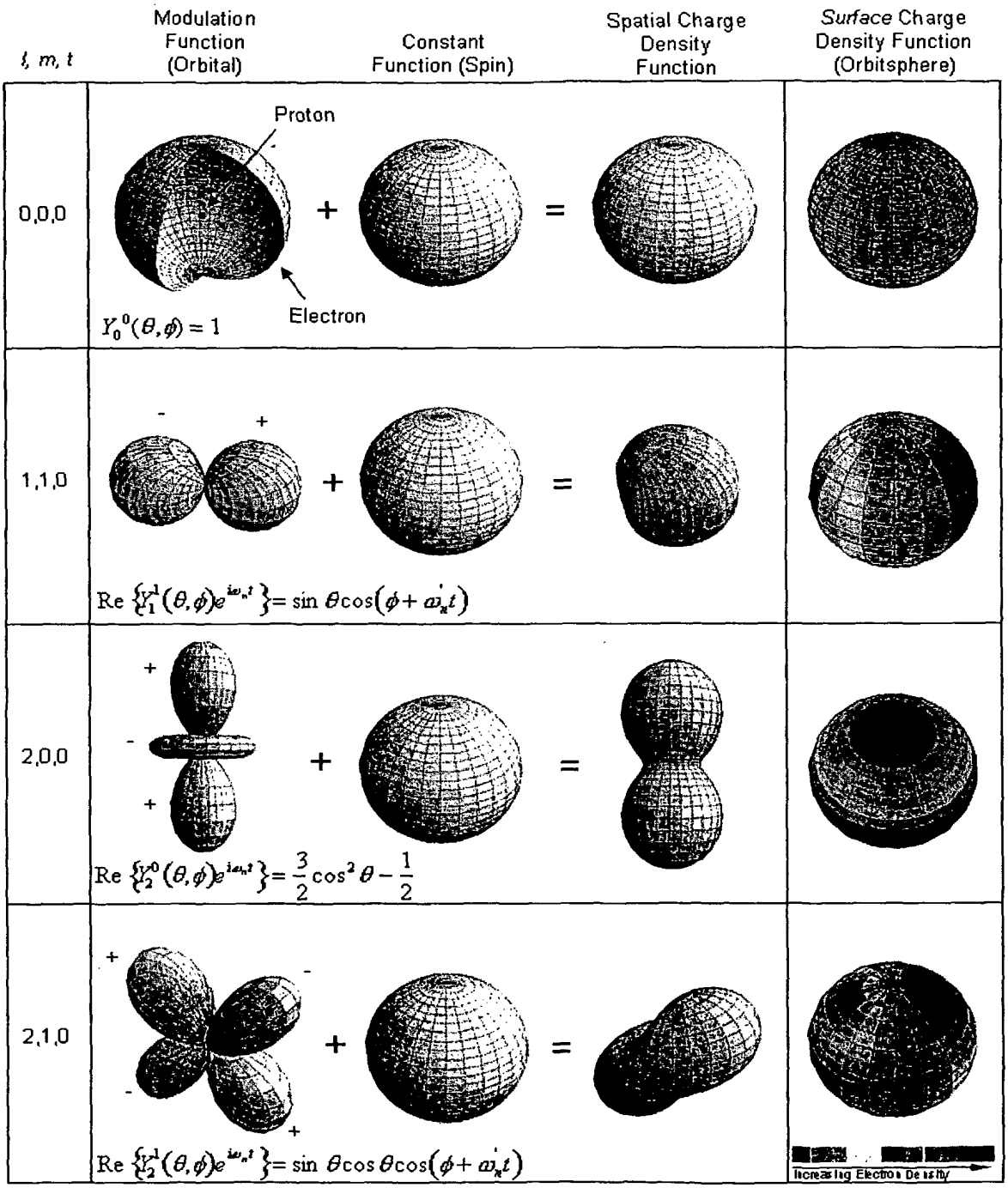


Fig. 3


l, m, t
Modulation Function (Orbital)
Constant Function (Spin)
Spatial Charge Density Function
Surface Charge Density Function (Orbitsphere)
0,0,0
Proton
Electron
$Y_0^0(\theta,\phi) = 1$
1,1,0
$\mathrm{Re}\left\{Y_1^{-1}(\theta,\phi)e^{i\omega_n t}\right\} = \sin\theta\cos(\phi + \omega_n' t)$
2,0,0
$\mathrm{Re}\left\{Y_2^0(\theta,\phi)e^{i\omega_n t}\right\} = \frac{3}{2}\cos^2\theta - \frac{1}{2}$
2,1,0
$\mathrm{Re}\left\{Y_2^{-1}(\theta,\phi)e^{i\omega_n t}\right\} = \sin\theta\cos\theta\cos(\phi + \omega_n' t)$
Increasing Electron Density

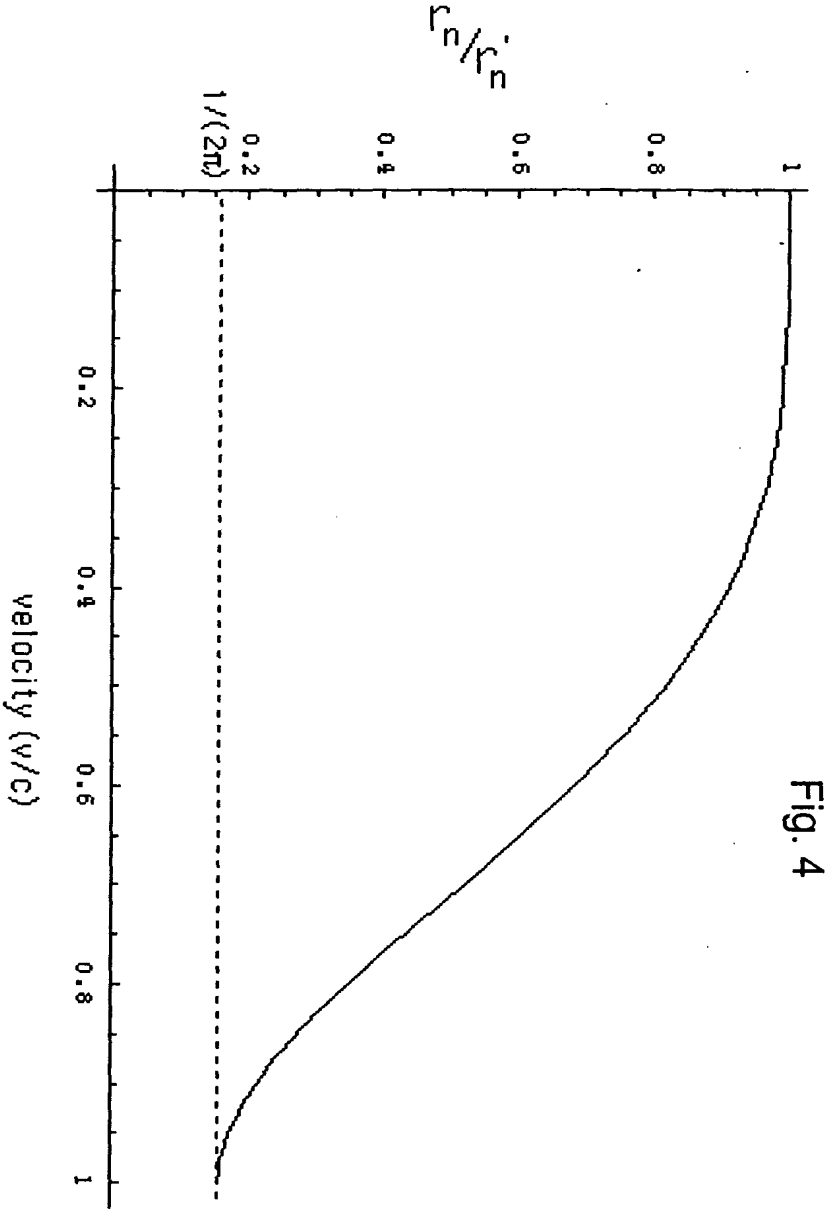


Fig. 4

Fig. 5